NONPARAMETRIC COMPARATIVE REVEALED RISK AVERSION

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ABSTRACT

We introduce a nonparametric method to compare risk aversion of different investors based on revealed preference methods. Using Yaari's (1969) definition of "more risk averse than", we show that it is sufficient to compare the revealed preference relations of two investors. This makes the approach operational; the central rationalisability theorem provides strong support for this approach. The approach is an alternative or complement to parametric approaches and a robustness check. As a necessary first step towards this comparative approach we show how to test data for consistency with stochastic dominance relations, which can also be used to recover larger parts of preferences. We include an application to experimental data by Choi et al. (2007) which demonstrates the potential of the comparative approach.

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1 INTRODUCTION

1.1 *Overview*

How do we measure the risk aversion of a decision maker—call him the "investor"—based on what we know about his preferences? Suppose we are given an investors utility function over monetary payoffs in different states of nature which occur with known probabilities. Then we can argue that if the set of risky gambles—call them "portfolios"—which investor A prefers over some status quo is a subset of the gambles which investor B prefers over the same status quo, then investor A is more risk averse than investor B. After all, there are risky gambles which investor B is willing to accept but investor A is not, but there are no risky gambles which investor A is willing to accept but investor B is not. This is, basically, Yaari's (1969) definition of "more risk averse than".

This is a very nice concept for theoretical treatments. For example, it is useful to analyse the ordering of various classes of utility functions in terms of risk aversion (see, for example, Bommier et al. 2012). But it is not operational in the sense that we usually do not observe an investor's true utility function. What is observable, however, and what we use as the primitives in this paper, are choices made by investors. The question is then how these observations can be used to compare the risk aversion of two investors. A parametric approach could, for example, consist in specifying a particular utility function, deriving its demand function for each set of offered portfolios, and then estimating the parameters of the demand function to learn about the parameters of the underlying utility function. The usefulness of this approach depends, among other things, on how well the specified utility function fits the data.

This paper takes a different, nonparametric approach, which is very robust in the sense that it does not depend on specifying a particular form of utility function. It uses choice data of investors to derive revealed preference relations, shows how to test these relations for certain behavioural assumptions, and how to use these relations to recover everything that can be said about the investor's preference without recovering anything that cannot be said. This is a necessary step for the main contribution of this paper: We use it to show how to compare the degree of risk aversion revealed by the choices of an investor with the risk aversion revealed by any other investor. We can also compare the revealed risk aversion of an investor with the risk aversion expressed in the form of a particular utility function. As it turns out, there are easily testable conditions on the observations on an investor.

The variant of Yaari's (1969) definition of "more risk averse than" which is employed here states that investor A is partially more risk averse than investor B if there are at least two portfolios x and y, where x has a higher expected value than y, and A prefers y over x while B prefers x over y. A necessary first step towards this comparative analysis is to test if the choices could indeed be the made by a risk averse utility maximising investor. This is because the definition is based on the idea that preferring a portfolio with a lower expected value over a portfolio with a higher expected value signifies risk aversion. A risk seeking investor might prefer a portfolio with a lower expected value precisely because it is more risky. In Section 3 and A.1 we argue that this is indeed the appropriate way to define risk aversion in our context.

For this necessary first step, we construct first or second order stochastic dominance (FSD or SSD) relations. Given the environment in which the investor makes his choices, these relations are known a priori and we can easily compute the set of all possible portfolios which have FSD or SSD over any particular portfolio. We then show how these relations can be combined with revealed preference relations, and how this allows us to test if an investor prefers portfolios which have FSD or SSD over other portfolios. The axiom derived for SSD is shown to be a necessary and sufficient condition for risk aversion.

The central contribution of this paper is the comparative approach. The test for consistency with Ssd is a straightforward extension of standard revealed preference analysis and included because it is a necessary first step for the comparative analysis. However, the analysis is of some interest in itself, as it provides a way to test for risk aversion. It also allows to recover more about an investor's preference relation and therefore complements the revealed preference analysis of choice under uncertainty.

In the comparative analysis, we may find that neither of two investors is more risk averse than the other. Then either (i) they have very similar preferences, or (ii) their extent of risk aversion is different for different income ranges, or (iii) they act according to distinct notions of risk aversion. Case (i) is a helpful result to classify two investors as belonging to the same category of risk preferences, as we cannot reject the hypothesis that the two investors have the same risk preferences. Cases (ii) and (iii) highlight the problem with a "one size fits all approach"; in particular, they show that comparisons based on parameter estimates rely on the specified form of the utility function.

The approach is illustrated with an application to the experimental data of Choi et al. (2007a). The data is tested for consistency with Ssd, which is confirmed for most subjects, based on the Afriat Efficiency Index (or Critical Cost Efficiency Index) supported by Monte-Carlo simulations. The comparative risk aversion approach is then applied to the data. We find that most experimental subjects are indeed comparable. If neither of two subjects is more risk averse than the other, we find that in the used sample this is mostly because they have similar preferences, i.e. they are classified as case (i) above.

The analysis provides a strong test of robustness for conclusions based on parameter estimates. Furthermore, while the nonparametric approach does not give a distribution of parameters of risk aversion in a population, it nonetheless allows to characterise the distribution of risk attitudes: The nonparametric approach tells us what percentage of the population is less or more risk averse than *any* given preference. This is illustrated by comparing the choices of subjects with several parameters of a utility function estimated by Choi et al. (2007a).

It is the combination of several strands of the literature that distinguishes the approach in this paper. The theoretical literature on risk preferences, choice under uncertainty, and comparative risk aversion is combined with the nonparametric analysis based on operational revealed preference theory. This combination can—and indeed is—applied to data. It is not claimed that the nonparametric approach should replace other approaches. The analysis here complements them and should, at the very least, be applied before further steps are taken, as it allows to draw strong conclusions about preferences without the need of restrictive assumptions on functional form.

1.2 Related Literature

This paper is related to the theoretical literature on choice under uncertainty and the discussion of what "risk" is, the comparative risk aversion literature, the revealed preference approach and the nonparametric analysis of choice data within consumer demand theory, and the experimental literature on risk preferences by subjects who are asked to make properly incentivised choices under controlled conditions.

Rothschild and Stiglitz (1970, 1971) provide a definition of "risk" and analyse its economic consequences. In particular, they call a random variable y "more variable" than a random variable x if x is equal to y plus a disturbance term with expected value of 0. Then y is a mean preserving spread (Mps) of x, and x has second order stochastic dominance over x. For two random variables with the same mean, they show that every element u in the set of all concave utility functions yields $u(y) > u(x)$ if and only if x is an Mps of y. Defining risk aversion in terms of second order stochastic dominance is therefore the least restrictive reasonable definition.

Similarly, Hadar and Russell (1969) note that comparing uncertain prospects in terms of moments is problematic if the utility function of an investor is not known. They define dominance of portfolios in terms of first- and second order stochastic dominance and show that any increasing utility function yields $u(y) > u(x)$ if and only if y has FSD over x, and any increasing concave utility function yields $u(y) > u(x)$ if and only if y has SSD over x. See also the early contribution of Hanoch and Levy (1969) in the same year with similar results, and Levy (1992) for a survey.

Yaari (1969) answers the question of when an investor A is more risk averse than B within a framework with one risky asset. Any investment in the risky asset is a gamble, and the acceptance set is the set of all gambles which are preferred to the status quo by an investor. Yaari suggests to call investor A more risk averse than investor B if the acceptance set of A is contained in the acceptance set of B. Similar approaches to uncertainty and ambiguity aversion are developed by Epstein (1999), Ghirardato and Marinacci (2002), and Grant and Quiggin (2005)

A seminal article by Pratt (1964), and similarly the work by Kihlstrom and Mirman (1974), analyses a measure of risk aversion based on certainty equivalents. In a recent paper, Bommier et al. (2012) provide a formal framework for analysing comparative risk aversion of different investors, with a focus on intertemporal choice. They use their approach to analyse several classes of utility functions common in the literature.

In the revealed preference approach it is assumed that the researcher observes a finite set of alternatives a decision maker has and the alternative which he actually chooses. The data are then used to construct the revealed preference relation. An advantage of the approach is that we do not need to assume any particular functional form of utility; the revealed preference approach therefore lends itself to a nonparametric analysis of choice data. Afriat's (1967) analysis, for example, makes the revealed preference approach operational when the sets of alternatives are competitive budget sets. Varian (1982, 1983a) refines this approach and provides highly valuable tools for the nonparametric analysis of such data. Clark (2000) considers the problem of recovering expected utility from observed choice behaviour, but does not provide extensive tools for the analysis of revealed preference data.

Varian (1983b) provides a condition which is necessary and sufficient for the existence of an expected utility function which rationalises a set of investment decisions. His condition is expressed as a linear feasibility system which has to have a solution. He applies his framework to a mean variance model of utility maximisation. The approach described here is more directly rooted in the axiomatic revealed preference approach and shows how to enrich revealed preference relations with FSD- and SSD-relations, and the recovered preferred and worse sets are shown to be useful for comparative risk aversion.

Experimental economics allows researchers to collect choice data of subjects under controlled conditions. "Induced budget experiments", where subjects are asked to make choices on competitive budget sets, are increasingly common.[1] Such experiments allow to collect extensive data on individuals' preference. Choi et al. (2007a), in particular, collect fifty decisions of each of ninety three subjects in an induced budget experiment on choice under uncertainty. They test the data for consistency with GARP. Furthermore, they estimate parameters of utility functions to characterise the distribution of risk preferences.

1.3 Outline

The rest of the paper is organised as follows: Section 2.1 introduces the framework and the notation. Section 2.2 reviews the necessary revealed preference literature and extends the approach using stochastic dominance

[1] See, for example, Sippel (1997), Harbaugh et al. (2001), Andreoni and Miller (2002), Février and Visser (2004), Chen et al. (2006), Choi et al. (2007a), Fisman et al. (2007), Banerjee and Murphy (2011).

relations. It derives the FSD-GARP and SSD-GARP, both of which are testable and which correspond to Varian's (1982) Generalised Axiom of Revealed Preference (GARP). In particular it is shown that SSD-GARP is necessary and sufficient for the existence of a monotonically increasing and concave utility function which rationalises the observations and which obeys second order stochastic dominance; SSD-GARP is therefore a necessary and sufficient condition for risk aversion. Section 3 introduces the nonparametric approach to compare the extent of risk aversion of two investors. Section 4 applies the methods to the experimental data of Choi et al. (2007a). Section 5 discusses the results and concludes. All proof can be found in the appendix in Section A.

2 THEORY: PRELIMINARIES

2.1 *Basic Definitions*

A set of observed investment choices consists of a set of chosen portfolios of assets and the prices and incomes at which these assets were chosen.[2] The asset space is $\mathbb{R}^L_+$ and the price space is $\mathbb{R}^L_{++}$, where $L \geq 2$ denotes the number of different assets.[3] Investors choose portfolios $x^i = (x^i_1, \dots, x^i_L)' \in \mathbb{R}^L_+$ of asset quantities when facing an asset price vector $p^i = (p^i_1, \dots, p^i_L) \in \mathbb{R}^L_{++}$; these choices are the demand we observe. A budget set is then defined by $B^i = B(p^i) = \{x \in \mathbb{R}^L_+ : p^i x \leq 1\}$; we will sometimes refer to a budget using the characterising price vector. The entire set of N observations on an investor is denoted as $\Omega = \{(x^i, p^i)\}_{i=1}^N$. We assume that demand is exhaustive (i.e., $p^i x^i = 1$). Let intB^i denote the interior of B^i.

There are L different states which can obtain after the portfolio choice has been made. In each state $i = 1, \dots, L$, asset i is the only asset that pays off. State i occurs with probability $\pi_i \in \Delta(L)$, where $\Delta(L)$ is the $(L-1)$ probability simplex, i.e., $\pi_i \geq 0$ for all i and $\sum_{i=1}^L \pi_i = 1$. The probability vector π is known to the investor and the observing researcher. To summarise the information used in this framework: We observe the number of possible states, the probabilities with which each of these states occurs, and N choices made by an investor for N different price vectors of the assets. Strictly speaking, we also need to assume that we observe the amount of money invested, which is used to normalise the price vector of the assets such that $p^i x^i = 1$. Given that we observe the possible states and the probability vector, stochastic dominance relations are known by definition (see below).

Note that the asset space $\mathbb{R}^L_+$ and the space of state contingent payoffs are the same. A portfolio $x = (x_1, \dots, x_L)$ specifies the amounts invested in L different assets, where an asset is a state-contingent claim. We can define an asset as a column vector $X_{\cdot,i} = (X_{1,i}, \dots, X_{L,i})'$ which specifies the payoff in the different states $1, \dots, L$, and x_i is the amount of money invested in this asset. In the present framework, asset i is simply given by $X_{i,i} = 1$ and $X_{j,i} = 0$ for $j \neq i$, and X is the identity matrix. These basic assets are also known as Arrow-Debreu securities. The payoff in state j of a portfolio x is then $(X_{j,\cdot})x = x_j$. Instead of defining investors' preferences over payoffs in the different states, we can equivalently define the preferences over portfolios.

This setup is much more general than it may appear: Suppose that instead of Arrow-Debreu securities, there are $K \geq 2$ linearly independent general assets $Y_{\cdot,j}$. Asset $Y_{\cdot,i}$ pays off $Y_{j,i} \geq 0$ in state j. If we allow short-selling of these assets, that is, to invest in a negative amount of some of the general assets, we need a no arbitrage (no free lunch) condition. If this condition holds, and if there are at least $K = L$ linearly

[2] "Portfolios" correspond to the term "lotteries".

[3] We use the following notation: For all $x, y \in \mathbb{R}^L$, $x \geqq y$ if $x_i \geq y_i$ for all $i = 1, \dots, L$; $x \geq y$ if $x \geqq y$ and $x \neq y$; $x > y$ if $x_i > y_i$ for all $i = 1, \dots, L$. We denote $\mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_i \geqq (0, \dots, 0)\}$ and $\mathbb{R}^L_{++} = \{x \in \mathbb{R}^L : x > (0, \dots, 0)\}$.

independent general assets, then the problem of choosing a portfolio of general assets is isomorphic to a problem of choosing a portfolio of Arrow-Debreu securities. Note that we can even account for safe assets, that is, assets which will pay the same amount no matter what state occurs: This is simply a general $Y_{\cdot,i}$ with $Y_{j,i} = c > 0$ for all states j. Thus, the assumption that demand is exhaustive ($p^i x^i = 1$) is justified; we do not need to account for the possibility that the investor only invests a part of his wealth in risky assets.

Then if instead of choices over basic Arrow-Debreu securities we observe choices over more general assets, we can transform the observations into an equivalent set of observations over (fictional) Arrow-Debreu securities. This follows from the work of Ross (1978), Breeden and Litzenberger (1978), and Varian (1987), among others. The appendix (Section A.1) contains a more detailed exposition of these facts. We carry out the analysis in terms of Arrow-Debreu assets as it simplifies the notation, allows to specify preferences directly over portfolios, and corresponds to the experiments conducted by Choi et al. (2007a).

We assume that an investor can be represented by transitive, complete, and continuous binary relation[4] on $\mathbb{R}^L_+$. This binary relation $\succsim \in \mathbb{R}^L_+ \times \mathbb{R}^L_+$ represents his *preference* according to which he decides which portfolio to choose on a budget. The interpretation is as usual, i.e. $(x, y) \in \succsim$, also written $x \succsim y$, means that to the investor x is at least as good as y. For $\succsim$ (and similarly for all other complete relations defined below) $\succ$ denotes the asymmetric part of $\succsim$ and $\sim$ denotes the symmetric part, i.e., $x \succ y$ if $x \succsim y$ and [not $y \succsim x$], and $x \sim y$ if $x \succsim y$ and $y \succsim x$.

For a given probability vector π, let $\mathbf{E}(x) = \sum \pi_i x_i$ be the expected value of a portfolio $x \in \mathbb{R}^L_+$. For convenience, we define the relation $\succsim_{\text{E}} \in \mathbb{R}^L_+ \times \mathbb{R}^L_+$ as

$$x \succsim_{\text{E}} y \text{ if } \mathbf{E}(x) \geq \mathbf{E}(y).$$

Let $\sim_{\text{E}}$ and $\succ_{\text{E}}$ denote the symmetric and asymmetric part of $\succsim_{\text{E}}$, respectively.

Let $\Pi(x)$ be the ex post payoff of the portfolio x. For a given π, let $F : \mathbb{R} \times \mathbb{R}^L_+ \to [0,1]$ be the cumulative distribution function of a portfolio, i.e., $F(\xi, x) = \text{Prob}(\Pi(x) \leq \xi)$ is the probability that the payoff from a portfolio $x \in \mathbb{R}^L_+$ is less than or equal to $\xi \in \mathbb{R}$. Let $\xi^i \in \mathbb{R}_+$, for $i = 1, \ldots, n \leq 2L$, be one of the payoffs of two portfolios x and y, i.e., $\xi^i \in \{x_1, \ldots, x_L\} \cup \{y_1, \ldots, y_L\}$, sorted in increasing order, with n denoting the number of distinct x_i and y_i. That is, when we compare any two portfolios x and y, an ξ^i is one of the ex post payoffs; with two portfolios there is at least one distinct ex post payoff and there are at most $2L$ distinct payoffs. Then let $\succsim_{\text{FSD}}$ and $\succsim_{\text{SSD}}$ be binary relations on $\mathbb{R}^L_+$, defined as

$$x \succsim_{\text{FSD}} y \text{ if } F(\xi^i, x, \pi) \leq F(\xi^i, y, \pi) \text{ for all } \xi^i$$

and

$$x \succsim_{\text{SSD}} y \text{ if } \sum_{i=1}^{\ell} F(\xi^i, x, \pi)[\xi^{i+1} - \xi^i] \leq \sum_{i=1}^{\ell} F(\xi^i, y, \pi)[\xi^{i+1} - \xi^i] \text{ for all } \ell < n \text{ and } \xi^i.$$

The relations are called the *first and second order stochastic dominance* relations, respectively (see Hadar and Russell 1969): x hast *first order stochastic dominance* (FSD) over y if $x \succsim_{\text{FSD}} y$, and *second order stochastic dominance* (SSD) if $x \succsim_{\text{SSD}} y$. Suppose x has FSD (SSD) over y. Then every expected utility maximiser with a monotonically increasing (and concave) utility function will prefer x over y (see, for example, Hanoch

[4]A binary relation $\succsim$ is *transitive* if $[x \succsim y$ and $y \succsim z]$ implies $x \succsim z$; it is *complete* if for every two bundles x, y, either $x \succsim y$ or $y \succsim x$; it is *continuous* if for all x the sets $\{y : x \succsim y\}$ and $\{y : y \succsim x\}$ are closed.

and Levy 1969). If $x \succsim_{\text{SSD}} \cap \sim_{\text{E}} y$, that is, x has SSD over y and the same expected value, then y is a *mean preserving spread* (MPS) of x, and x is a *mean preserving contraction* (MPC) of y.

Axiom 1 *A preference $\succsim$ satisfies the* Axiom of First Order Stochastic Dominance *(AFSD) if* $\succsim_{\text{FSD}} \subset \succsim$. *A preference $\succsim$ satisfies the* Axiom of Second Order Stochastic Dominance *(ASSD) if* $\succsim_{\text{SSD}} \subset \succsim$.

Note that ASSD $\Rightarrow$ AFSD but not vice versa. We will also say that investors whose preferences satisfy AFSD or ASSD are FSD-rational or SSD-rational.

For the following analysis, it is convenient to describe the set of all portfolios which have second order stochastic dominance over some reference portfolio x. Because we will also need the set of all portfolios which are ranked above some x by other binary relations, we define a more general set, which conveniently depends on some arbitrary binary relation Q. For a given π, let

$$\mathcal{P}(x, \text{Q}) = \{y \in \mathbb{R}^L_+ : y \, \text{Q} \, x\}.$$

Then $\mathcal{P}(x, \succsim_{\text{SSD}})$ is the set of all portfolios which have SSD over x, and $\mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$ is the set of portfolios which have SSD over x and the same expected value as x. (i.e., the set of all MPCs of x).

We record a first lemma to be used later but independently worth mentioning.

Lemma 1 *The relation $\succsim_{SSD}$ is quasi-concave, i.e., $\mathcal{P}(x, \succsim_{SSD})$ is a convex set for all $\pi \in \Delta(L)$.*

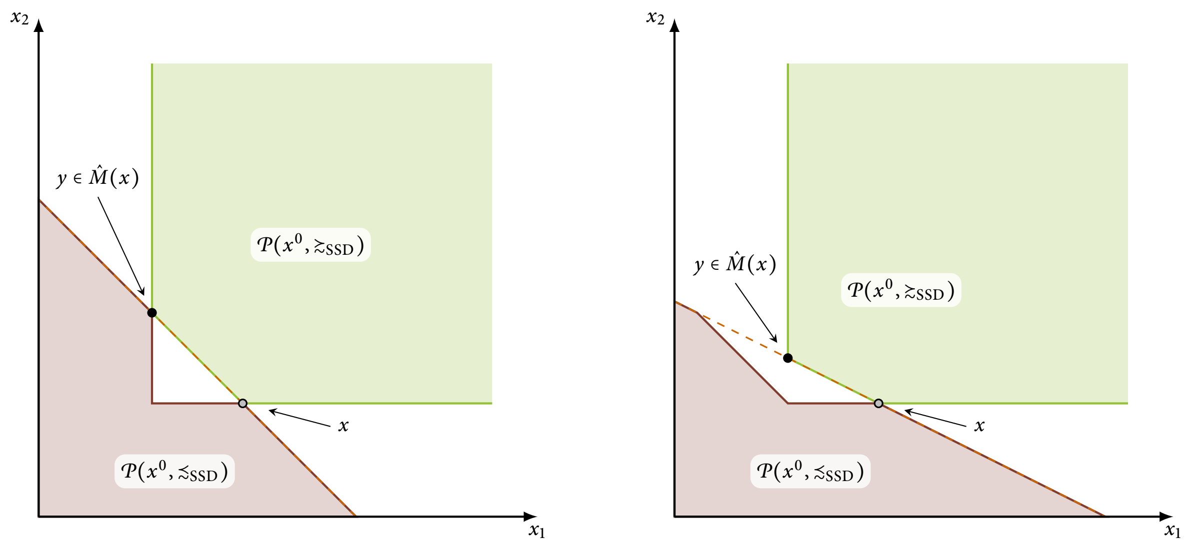
The *convex hull* CH of a set of points $Y = \{y^i\}$ and its *convex monotonic hull* CMH are defined as

$$\text{CH}(Y) = \Big\{x \in \mathbb{R}^L_+ : x = \sum_i \lambda_i y^i, \lambda \geq 0, \sum_i \lambda_i = 1\Big\}$$
$$\text{CMH}(Y) = \text{interior of CH}\Big(\big\{x \in \mathbb{R}^L_+ : x \geqq y^i \text{ for some } i\big\}\Big),$$

and $\overline{\text{CMH}}$ is the closure of CMH. Again, we will later need the convex monotonic hull of a set of portfolios which are ranked higher than x by some binary relation Q. Thus, for some Q on $\mathbb{R}^L_+$ we also write $\text{CMH}(x, \text{Q}) = \text{CMH}\left(\{y \in \mathbb{R}^L_+ : y \, \text{Q} \, x\}\right)$.

Before we turn to the necessary definitions for our rationalisability results below, we consider the two simple examples with $L = 2$ in Figure 1 (ignore the indicated $\hat{M}$ for now) to illustrate the set $\mathcal{P}(x, \succsim_{\text{SSD}})$ and the usefulness of the convex monotonic hull. In Figure 1.(a), the probability vector is $\pi = (½, ½)$; suppose that the indicated portfolio x^0 is $(x_1, x_2) = (45, 25)$. Clearly, in terms of stochastic dominance, the portfolio $(x_2, x_1) = (25, 45)$ must be considered equivalent to the portfolio x, as both portfolios have the same expected value and the same cumulative distribution function. It is also clear that every portfolio which consists of a convex combination of $(45, 25)$ and $(25, 45)$ is an MPC of x. Furthermore, portfolios which dominate an MPC of x (i.e., portfolios which pay off at least the same amount as the MPC in both states and more in at least one state), have SSD over x. Thus, every portfolio in the the convex monotonic hull of the set of all MPCs of x must have SSD over x. One can show that this relation also holds in the opposite direction, that is, $\mathcal{P}(x, \succsim_{\text{SSD}}) = \text{CMH}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$ (see Lemma 2 below). The set $\mathcal{P}(x, \precsim_{\text{SSD}})$, which is also shown in the figure, is the set of all portfolios over which x has SSD; note that $z \in \mathcal{P}(x, \precsim_{\text{SSD}})$ if and only if $x \in \mathcal{P}(z, \succsim_{\text{SSD}})$.

If the probabilities for each state are the same, the problem remains simple for $L > 2$: $\mathcal{P}(x, \succsim_{\text{SSD}})$ will always be the convex monotonic hull of all permutations of x. The problem becomes more complicated when we consider different probabilities, as in Figure 1.(b) with $\pi = (⅓, ⅔)$. Suppose that $x = (45, 25)$; then we need to find an MPC y of x such that y_2 is maximal in order to describe the set of all MPCs of x as a convex combination of two portfolios. This y can be shown to be $(25, 35)$. What remains the same in comparison with the example with $\pi = (½, ½)$, and what will turn out to be generally true, is that $\mathcal{P}(x, \succsim_{\text{SSD}})$ is the convex monotonic hull of all MPCs of x.



(a) (b)

Figure 1: Example with probabilities $(\pi_1, \pi_2) = (½, ½)$ (a) and $(\pi_1, \pi_2) = (⅓, ⅔)$ (b). The dashed line shows all portfolios with the same expected value as the portfolio x. Both figures show the set of portfolios which have second order stochastic dominance over x, and the set of portfolios over which x has second order stochastic dominance.

The following definition is somewhat cumbersome, but it necessary for our purposes. We give several examples below to illustrate it. It generalises the two examples in Figure 1. Define recursively for some sequence of indices $\{i_j\}_{j=1}^n$, $n \leq L-1, 1 \leq i_j \leq L$,

$$M(x, \{i_1\}) = \{y \in \mathbb{R}_+^L : y = \arg\max_{\{\tilde{y} \in \mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})\}} \tilde{y}_{i_1}\},$$
$$M(x, \{i_j\}_{j=1}^n) = \{y \in \mathbb{R}_+^L : y = \arg\max_{\{\tilde{y} \in M(x, \{i_j\}_{j=1}^{n-1})\}} \tilde{y}_{i_n}\}.$$

Let $\hat{M}(x)$ denote the union of all $M(x, \{i_j\}_{j=1}^{L-1})$ for every permutation of indices from 1 to L.

The understand the construction of M, consider first the two dimensional case ($L = 2$). Consider the set of portfolios which have the same expected value as x and have SSD over x (i.e., $\mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$). Of these portfolios, $M(x, \{1\})$ and $M(x, \{2\})$ select the ones that have the maximal payoff in state 1 and 2, respectively. Note that $M(x, \{i\})$, $i = 1, 2$, are singletons, and one of these sets contains x if $x_1 \neq x_2$; if $x_1 = x_2$, $M(x, \{1\}) = M(x, \{2\}) = x$. This is shown in both parts of Figure 1 for the portfolio x.

For $L = 3$, $M(x, \{1\})$ again selects the set of all portfolios in $\mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$ which have the maximal payoff in state 1; here, $M(x, \{1\})$ is not necessarily a singleton. Then, $M(x, \{1, 2\})$ selects the one portfolio in $M(x, \{1\})$ which has the maximal payoff in state 2. One more example for $L = 4$: $M(x, \{1, 4, 2\})$ selects the one portfolio in $M(x, \{1, 4\})$ which has the maximal payoff in state 2 (i.e., take the set of portfolios in $\mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$ which have the maximum payoff in state 1; of those take those which have the maximum payoff in state 4; of those take the portfolio which has the maximum payoff in state 2). Note that $M(x, \{i_j\}_{j=1}^{L-1})$ is always a singleton.

By construction $y \in M(x, \{i_j\}_{j=1}^{L-1})$ is an Mpc of x, and x is an Mps of y. The intuition which we developed above can now be generalised in the following Lemma 2.

Lemma 2 *For all $x \in \mathbb{R}_+^L$,*

(i) $\mathcal{P}(x, \succsim_{SSD} \cap \sim_E) = CH(\hat{M}(x))$,

(ii) $\mathcal{P}(x, \succsim_{SSD}) = \overline{CMH}(x, \succsim_{SSD} \cap \sim_E)$, *and thus* $\mathcal{P}(x, \succsim_{SSD}) = \overline{CMH}(\hat{M}(x))$.

2.2 *Revealed Preference*

Revealed preference relations, like preferences, are binary relations on $\mathbb{R}_+^L$ which we observe due to an investor's choices $\Omega = \{(x^i, p^i)\}_{i=1}^N$ combined with theoretical reasoning about what these choices reveal. Let $\text{Q} \subseteq \mathbb{R}_+^L \times \mathbb{R}_+^L$ be any binary relation. Then the *transitive closure* $(\text{Q})^+$ of Q is defined as the smallest transitive relation that contains Q, that is, $x(\text{Q})^+ y$ if there are $x', \dots, x'''$ such that $x \,\text{Q}\, x'$, $x' \,\text{Q}\, x''$, …, $x''' \,\text{Q}\, y$. We the use the following definitions to recover an investor's preference that is implicit in a set of portfolio choices:

- The portfolio x^i is *directly revealed preferred* to a portfolio x, written $x^i \,\text{R}^0\, x$, if $p^i x^i \geq p^i x$.
- The portfolio x^i is *strictly directly revealed preferred* to a portfolio x, written $x^i \,\text{P}^0\, x$, if $p^i x^i > p^i x$.
- Let $\text{R} = (\text{R}^0)^+$. Then the portfolio x^i is *revealed preferred* to a portfolio x if $x^i \,\text{R}\, x$.
- The portfolio x^i is *strictly revealed preferred* to a portfolio x, written $x^i \,\text{P}\, x$, if for some sequence of observations $x^i \,\text{R}\, x^j, x^j \,\text{P}^0\, x^k, x^k \,\text{R}\, x$.

Axiom 2 (Varian 1982) *A set of observations Ω satisfies the* Generalised Axiom of Revealed Preference *(Garp) if* [*not* $x^i \,\text{P}^0\, x^j$] *whenever* $x^j \,\text{R}\, x^i$.

The strength of Garp is based on the fact that it is an easily testable condition and is a necessary and sufficient condition for utility maximisation, as Afriat's Theorem demonstrates. We say that a utility function $u : \mathbb{R}_+^L \to \mathbb{R}$ *rationalises* a set of observations Ω if $u(x) \geq u(y)$ whenever $x \,\text{R}\, y$. Let $\mathcal{U}$ denote the set of all continuous, non-satiated, monotonic, and concave utility functions.

Theorem 1 (Afriat 1967, Diewert 1973, Varian 1982) *The following conditions are equivalent:*

1. *there exists a $u \in \mathcal{U}$ which rationalises the set of observations Ω;*
2. *the set of observations Ω satisfies Garp.*

The revealed preference relations can be extended by imposing axioms Afsd or Assd. If the hypotheses are correct, then R is the subset of some preference $\succsim$. If the investor's preference satisfies first order stochastic dominance, then $\succsim_{\text{FSD}}$ is a subset of the same preference $\succsim$. Thus $(\text{R} \cup \succsim_{\text{FSD}}) \subset \succsim$, and similarly for $\succsim_{\text{SSD}}$. Define

$$
\begin{aligned}
&\text{R}_{\text{FSD}} = (\text{R} \cup \succsim_{\text{FSD}})^+, \quad \text{R}_{\text{SSD}} = (\text{R} \cup \succsim_{\text{SSD}})^+, \quad \text{P}^0_{\text{FSD}} = \text{P}^0 \cup \succ_{\text{FSD}}, \quad \text{P}^0_{\text{SSD}} = \text{P}^0 \cup \succ_{\text{SSD}}, \\
&\text{P}_{\text{FSD}} = \{(x, y) \in \mathbb{R}_+^L \times \mathbb{R}_+^L : x \,\text{R}_{\text{FSD}}\, z \,\text{P}^0_{\text{FSD}}\, z' \,\text{R}_{\text{FSD}}\, y \text{ for some } z, z' \in \mathbb{R}_+^L\}, \\
&\text{P}_{\text{SSD}} = \{(x, y) \in \mathbb{R}_+^L \times \mathbb{R}_+^L : x \,\text{R}_{\text{SSD}}\, z \,\text{P}^0_{\text{SSD}}\, z' \,\text{R}_{\text{SSD}}\, y \text{ for some } z, z' \in \mathbb{R}_+^L\}.
\end{aligned}
\tag{1}
$$

Let $\sigma_\ell(x)$ denote the ℓth permutation of x, with $\sigma_1(x) = x$. Let $L!$ denote the factorial of L. Define

$$\sigma(\Omega) = \left\{y \in \mathbb{R}^L_+ : y = \sigma_\ell(x^i) \text{ for some } i = 1, \ldots, N \text{ and some } \ell = 1, \ldots, L!\right\}.$$

We will refer to the elements in $\sigma(\Omega)$ as s^i; the ith element of $\sigma(\Omega)$ will be denoted $\sigma(\Omega)^i$. Note that all $x^i \in \sigma(\Omega)$; let the set be sorted such that $\sigma(\Omega)^i = x^i$ for $i = 1, \ldots, N$. Define

$$\tau(\Omega) = \left\{y \in \mathbb{R}^L_+ : y \in \hat{M}(x^i) \text{ for some } i = 1, \ldots, N\right\}.$$

We will refer to the elements in $\tau(\Omega)$ as t^i. Again we have $x^i \in \tau(\Omega)$; let $\tau(\Omega)$ be sorted in the same way as $\sigma(\Omega)$.

Axiom 3 *A set of observations Ω satisfies the FSD-GARP if for all $s^i \in \sigma(\Omega)$,*

$$[\textit{not } s^i\, P_{FSD}\, s^j] \textit{ whenever } s^j\, R_{FSD}\, s^i.$$

It satisfies the SSD-GARP if for all $t^i \in \tau(\Omega)$,

$$[\textit{not } t^i\, P_{SSD}\, t^j] \textit{ whenever } t^j\, R_{SSD}\, t^i.$$

We say that a utility function u *FSD-rationalises* a set of observations Ω if $u(x) \geq u(y)$ whenever $x \, \mathrm{R}_{\mathrm{FSD}} \, y$; it *SSD-rationalises* Ω if $u(x) \geq u(y)$ whenever $x \, \mathrm{R}_{\mathrm{SSD}} \, y$.

Theorem 2 *The following conditions are equivalent:*

1. *there exists a $u \in \mathcal{U}$ which FSD-rationalises (SSD-rationalises) the set of observations Ω;*
2. *the set of observations Ω satisfies FSD-GARP (SSD-GARP).*

Note that Theorem 2 shows that SSD-GARP is a necessary and sufficient condition for risk aversion in the SSD-sense.

Following Varian (1982), we now turn to the question of recoverability of preferences. Given some portfolio $x^0 \in \mathbb{R}^L_+$ which was not necessarily observed as a choice, the set of prices which *support* x^0 is defined as

$$\begin{aligned} S(x^0) &= \left\{p^0 \in \mathbb{R}^L_{++} :\ \{(x^i, p^i)\}_{i=0}^N \text{ satisfies GARP and } p^0 x^0 = 1\right\}, \\ S_{\mathrm{FSD}}(x^0) &= \left\{p^0 \in \mathbb{R}^L_{++} :\ \{(x^i, p^i)\}_{i=0}^N \text{ satisfies FSD-GARP and } p^0 x^0 = 1\right\}, \\ S_{\mathrm{SSD}}(x^0) &= \left\{p^0 \in \mathbb{R}^L_{++} :\ \{(x^i, p^i)\}_{i=0}^N \text{ satisfies SSD-GARP and } p^0 x^0 = 1\right\}. \end{aligned}$$

Varian (1982) uses $S(x^0,)$ to describe the set of all bundles (here: portfolios) which are revealed worse and revealed preferred to a portfolio x^0: If for any price vector at which x^0 can be demanded without violating GARP x^0 must be revealed preferred to x, then x is in the set of all portfolios revealed worse to x^0, and similarly for revealed preferred sets. Thus, we can define the revealed preference analogy to $\mathcal{P}(x, \succsim)$: The set of all portfolios which are *revealed worse* than x^0 is given by

$$\mathcal{RW}(x^0, \mathrm{R}) = \left\{x \in \mathbb{R}^L_+ :\ \text{for all } p^0 \in S(x^0),\ x^0 \, \mathrm{P} \, x\right\}$$

and the set of all portfolios which are *revealed preferred* to x^0 is given by

$$\mathcal{RP}(x^0, \mathrm{R}) = \left\{x \in \mathbb{R}^L_+ : \text{ for all } p \in S(x),\ x\,\mathrm{P}\,x^0\right\}.$$

Similarly, we define

$$\mathcal{RW}(x^0, \mathrm{R}_{\mathrm{FSD}}) = \left\{x \in \mathbb{R}^L_+ : \text{ for all } p^0 \in S_{\mathrm{FSD}}(x^0),\ x^0\,\mathrm{P}\,x\right\},$$
$$\mathcal{RW}(x^0, \mathrm{R}_{\mathrm{SSD}}) = \left\{x \in \mathbb{R}^L_+ : \text{ for all } p^0 \in S_{\mathrm{SSD}}(x^0),\ x^0\,\mathrm{P}\,x\right\},$$

and

$$\mathcal{RP}(x^0, \mathrm{R}_{\mathrm{FSD}}) = \left\{x \in \mathbb{R}^L_+ : \text{ for all } p \in S_{\mathrm{FSD}}(x),\ x\,\mathrm{P}\,x^0\right\},$$
$$\mathcal{RP}(x^0, \mathrm{R}_{\mathrm{SSD}}) = \left\{x \in \mathbb{R}^L_+ : \text{ for all } p \in S_{\mathrm{SSD}}(x),\ x\,\mathrm{P}\,x^0\right\}.$$

These definitions are well motivated by the equivalence of Garp with the existence of a concave utility function which rationalises the data: *Any* utility function which rationalises a set of observations must have $u(x) > u(x^0)$ if $x \in \mathcal{RP}(x^0, \mathrm{R})$, etc. See Figure 2 for an example.

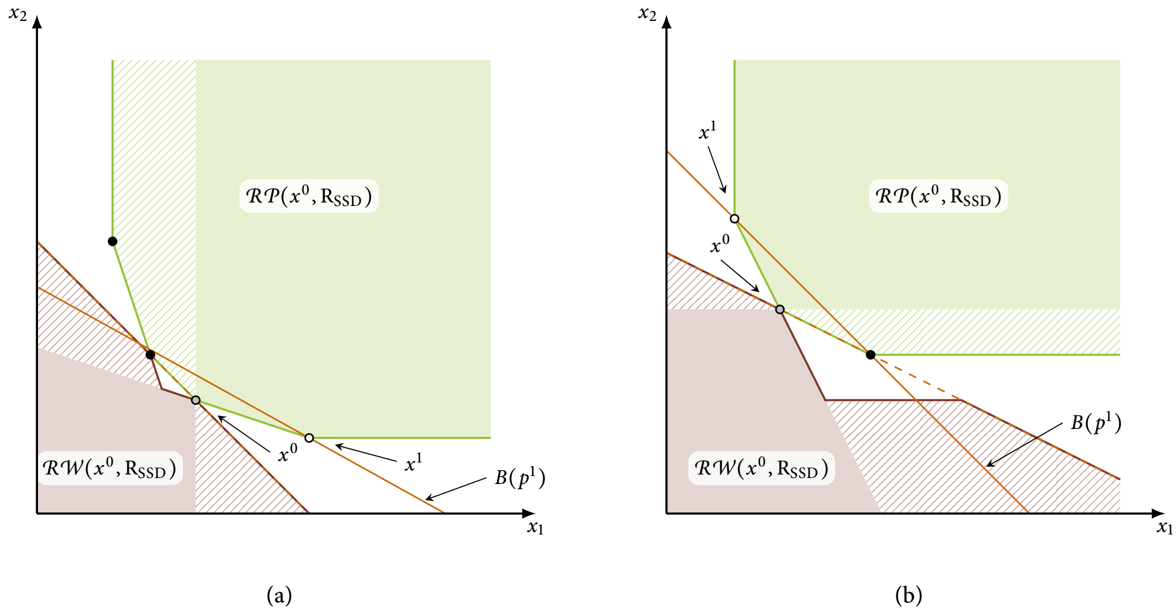


Figure 2: Example with probabilities $(\pi_1, \pi_2) = (½, ½)$ (a) and $(\pi_1, \pi_2) = (⅓, ⅔)$ (b). Revealed preferred and revealed worse set of x^0 with one observation (x^1, p^1), based on the extended relation $\mathrm{R}_{\mathrm{SSD}}$. The dashed regions show what is added by combining R and $\succsim_{\mathrm{SSD}}$.

The next proposition shows that we can express the $\mathcal{RP}$ sets conveniently as convex monotonic hulls of a finite set of points.

Proposition 1 *For all $x \in \mathbb{R}^L_+$, if the set of observations Ω satisfies*

(i) Garp, then $CMH(x^0, R) \subseteq \mathcal{RP}(x^0, R) \subseteq \overline{CMH}(x^0, R)$;

(ii) Fsd-Garp, then $CMH(x^0, R_{FSD}) \subseteq \mathcal{RP}(x^0, R_{FSD}) \subseteq \overline{CMH}(x^0, R_{FSD})$;

(iii) Ssd-Garp, then $CMH(x^0, R_{SSD}) \subseteq \mathcal{RP}(x^0, R_{SSD}) \subseteq \overline{CMH}(x^0, R_{SSD})$.

Varian (1982) and Knoblauch (1992) prove part (i) of the proposition. We omit the proof for the other parts, which are along the lines of Knoblauch's (1992) proof; Lemma 2 makes the extension quite simple.

3 THEORY: INTERPERSONAL COMPARISON

Let $\trianglerighteq \in \times_{i=1}^{4} \mathbb{R}_+^L$ be the *more risk averse than* relation. For two preferences $\underline{\succsim}$ and $\hat{\succsim}$ which satisfy AsSD (and therefore AFSD), define

$$\underline{\succsim} \trianglerighteq \hat{\succsim} \text{ if } [\hat{\succsim} \cap \prec_E] \subseteq [\underline{\succsim} \cap \prec_E].$$

That is, an investor $\underline{\succsim}$ is more risk averse than an investor $\hat{\succsim}$ if the set of portfolios with a lower expected value than x which are preferred to x by $\hat{\succsim}$ is a subset of the corresponding set of $\underline{\succsim}$. Clearly, this construction makes sense only if the two investors are not risk seeking, which is why we need the test for SSD-rationality of the previous section. Let $\triangleright$ be the asymmetric part of $\trianglerighteq$, that is, $\underline{\succsim}$ is *strictly more risk averse than* $\hat{\succsim}$, written $\underline{\succsim} \triangleright \hat{\succsim}$, if $\underline{\succsim} \triangleright \hat{\succsim}$ and [not $\hat{\succsim} \triangleright \underline{\succsim}$].

The definition of more risk averse is closely modelled on Yaari's (1969) concept, who considers acceptance sets of gambles. If investor A prefers all gambles over the status quo which investor B also prefers over the status quo, and there are additional gambles which A prefers but B does not, then B is more risk averse than A. The definition of $\trianglerighteq$ translates this concept to the framework considered here. Note that we do not claim that *preferring a higher expected value over lower expected value* is a sign a risk aversion, but that a risk averse utility maximiser who *prefers a lower expected value over a higher expected value* does so because the preferred portfolio is less risky. And if we observe choices of *two* investors, of which both are risk averse utility maximisers, and the first one strictly prefers a portfolio with a lower expected expected value over a portfolio with a higher expected value while the second one does not, then we conclude that the first one is at least partially more risk averse.

Note that given our definitions of budgets, an investor who has to choose a portfolio from a budget always has the option of choosing a riskless portfolio which pays off the same amount in all states. This riskless portfolio can be considered as the status quo in any situation where the investor has to choose a portfolio from a budget. Then the set of all other portfolios in the budget which the investor prefers to the riskless portfolio is the acceptance set. Any risk averse utility maximiser will only prefer portfolios over the riskless portfolio which are not stochastically dominated by the riskless portfolio; but then these portfolios must have a higher expected value than the riskless portfolio. We illustrate this in the Appendix A.1.

We will now consider two investors, on which we have sets of observations $\underline{\Omega}$ and $\hat{\Omega}$, and we will refer to these two investors by their revealed preference relations $\underline{\text{R}}$ and $\hat{\text{R}}$. Let $\underline{\mathcal{RP}}(x, \underline{\text{R}})$ and $\underline{\mathcal{RW}}(x, \underline{\text{R}})$ be the revealed preferred and worse set of the investor with the revealed preference relation $\underline{\text{R}}$, and analogously for $\hat{\text{R}}$. Let $\underline{\mathcal{RPL}}(x, \underline{\text{R}}) = \underline{\mathcal{RP}}(x, \underline{\text{R}}) \cap \mathcal{P}(x, \prec_E)$ and $\underline{\mathcal{RWL}}(x, \underline{\text{R}}) = \underline{\mathcal{RW}}(x, \underline{\text{R}}) \cap \mathcal{P}(x, \prec_E)$, and analogously for $\hat{\text{R}}$.

How can $\trianglerighteq$ be made operational given a finite set of observations on an investor and the revealed preference relation based on these observations? One problem is that R is only an incomplete relation, and therefore $x \notin \mathcal{RP}(x, \text{R})$ does *not* imply $x \in \mathcal{RW}(x, \text{R})$. Thus, we cannot base the statement that investor $\underline{\text{R}}$ is more risk averse than $\hat{\text{R}}$ on the fact that $\mathcal{R}\hat{\mathcal{P}}\mathcal{L}(x, \hat{\text{R}}) \subset \underline{\mathcal{RPL}}(x, \underline{\text{R}})$. This condition alone cannot exclude the possibility that the two investors make choices according to the complete preferences $\underline{\succsim}$ and $\hat{\succsim}$ such that $\mathcal{P}(x, \hat{\succsim}) \supseteq \mathcal{P}(x, \underline{\succsim})$. We therefore introduce a more careful concept: If, for some portfolio x, there is a y with a lower expected value than x which is preferred to x by investor $\underline{\text{R}}$, and at the same time investor $\hat{\text{R}}$ prefers

x to y, then investor $\underline{R}$ is at least partially more risk averse than $\hat{R}$. If $\underline{R}$ is partially more risk averse than $\hat{R}$, but $\hat{R}$ is not partially more risk averse than $\underline{R}$, then we conclude that $\underline{R}$ is more risk averse than $\hat{R}$.

Define $\trianglerighteq_{RA} \in \mathbb{R}^L_+ \times \mathbb{R}^L_+$ as

$$\underline{R} \trianglerighteq_{RA} \hat{R} \text{ if there exists } x \in \mathbb{R}^L_+ \text{ such that } \underline{\mathcal{RPL}}(x, \underline{R}) \cap \mathcal{R}\hat{\mathcal{W}}\mathcal{L}(x, \hat{R}) \neq \varnothing; \tag{2}$$

if $\underline{R} \trianglerighteq_{RA} \hat{R}$, we say that $\underline{R}$ is *partially revealed more risk averse than* $\hat{R}$. Then $\underline{R}$ is *revealed more risk averse than* $\hat{R}$, written $\underline{R} \triangleright_{RA} \hat{R}$, if $\underline{R} \trianglerighteq_{RA} \hat{R}$ and [not $\hat{R} \trianglerighteq_{RA} \underline{R}$].

Define

$$\delta(\underline{\Omega}, \hat{\Omega}) = \begin{cases} 1 & \text{if there are } \underline{x}^i \prec_E \hat{x}^j \text{ and } \left([\underline{x}^i \, \underline{R} \, \hat{x}^j \text{ and } \hat{x}^j \, \hat{P} \, \underline{x}^i] \text{ or } [\underline{x}^i \, \underline{P} \, \hat{x}^j \text{ and } \hat{x}^j \, \hat{R} \, \underline{x}^i]\right), \\ 0 & \text{otherwise,} \end{cases}$$

where $\underline{x}^i$ is a choice in $\underline{\Omega}$ and $\hat{x}^j$ is a choice in $\hat{\Omega}$.

The following theorem only considers data which satisfy the SSD-GARP. To see why, consider two portfolios x and y and let $L = 2$, $\pi = (1/3, 2/3)$, $x = (12, 0)$ and $y = (6, 6)$, such that $y \succ_E x$ and $y \succ_{SSD} x$. An investor may prefer x over y even though y has a higher expected value, but this cannot be the result of risk aversion. Such an investor can satisfy GARP, but not SSD-GARP, and his behaviour cannot (should not) be considered a sign of risk aversion.

Theorem 3 *Suppose $\underline{\Omega}$ and $\hat{\Omega}$ satisfy SSD-GARP.*

1. *The following conditions are equivalent:*
   - *(i)* $\delta(\underline{\Omega}, \hat{\Omega}) = 1$ *and* $\delta(\hat{\Omega}, \underline{\Omega}) = 0$;
   - *(ii)* $\underline{R}_{SSD} \triangleright_{RA} \hat{R}_{SSD}$;
   - *(iii) there exist* $\underline{u}, \hat{u} \in \mathcal{U}$ *which SSD-rationalise* $\underline{\Omega}$ *and* $\hat{\Omega}$*, respectively, and there do not exist* $\underline{v}, \hat{v} \in \mathcal{U}$ *which SSD-rationalise* $\underline{\Omega}$ *and* $\hat{\Omega}$*, respectively, such that for all* $x, y \in \mathbb{R}^L_+$ *with* $\mathbf{E}(x) < \mathbf{E}(y)$, $\hat{u}(x) > \hat{u}(y) \Rightarrow \underline{u}(x) > \underline{u}(y)$ *and* $\underline{v}(x) > \underline{v}(y) \Rightarrow \hat{v}(x) > \hat{v}(y)$.
2. *The following conditions are equivalent:*
   - *(i)* $\delta(\underline{\Omega}, \hat{\Omega}) = \delta(\hat{\Omega}, \underline{\Omega}) = 1$;
   - *(ii)* $\underline{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD}$ *and* $\hat{R}_{SSD} \trianglerighteq_{RA} \underline{R}_{SSD}$;
   - *(iii) there do not exist* $\underline{u}, \hat{u} \in \mathcal{U}$ *which SSD-rationalise* $\underline{\Omega}$ *and* $\hat{\Omega}$*, respectively, such that for all* $x, y \in \mathbb{R}^L_+$ *with* $\mathbf{E}(x) < \mathbf{E}(y)$, $\hat{u}(x) > \hat{u}(y) \Rightarrow \underline{u}(x) > \underline{u}(y)$ *or* $\underline{u}(x) > \underline{u}(y) \Rightarrow \hat{u}(x) > \hat{u}(y)$.
3. *The following conditions are equivalent:*
   - *(i)* $\delta(\underline{\Omega}, \hat{\Omega}) = \delta(\hat{\Omega}, \underline{\Omega}) = 0$;
   - *(ii)* [*not* $\underline{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD}$] *and* [*not* $\hat{R}_{SSD} \trianglerighteq_{RA} \underline{R}_{SSD}$];
   - *(iii) there exist* $\underline{u}, \hat{u} \in \mathcal{U}$ *and* $\underline{v}, \hat{v} \in \mathcal{U}$ *which SSD-rationalise* $\underline{\Omega}$ *and* $\hat{\Omega}$*, respectively, such that for all* $x, y \in \mathbb{R}^L_+$ *with* $\mathbf{E}(x) < \mathbf{E}(y)$, $\hat{u}(x) > \hat{u}(y) \Rightarrow \underline{u}(x) > \underline{u}(y)$ *and* $\underline{v}(x) > \underline{v}(y) \Rightarrow \hat{v}(x) > \hat{v}(y)$.

Theorem 3 is quite powerful: It shows that it is necessary and sufficient to compare only choices observed by one of the two investors, even though the definition of $\trianglerighteq_{RA}$ uses *all* $x \in \mathbb{R}^L_+$. The theorem therefore provides a nonparametric way to compare the risk aversion of two investors with only a finite number of comparisons. The third statement in the three parts of Theorem 3 provides strong support for the suggested definition of "revealed more risk averse than". For example, (1).(iii) shows that if we find that $\underline{R}_{SSD} \triangleright_{RA} \hat{R}_{SSD}$, then there exists utility functions $\underline{u}$ and $\hat{u}$ for the two investors which represent the investors choices such that, whenever y has a higher expected value than x and $\underline{u}$ prefers x over y, then so does $\hat{u}$. Furthermore, there does *not* exist a pair of utility function $\underline{v}$ and $\hat{v}$ such that whenever $\hat{v}$ prefers x over y, so does $\underline{v}$.

We say that two investors are (a) *similar* if [not $\underline{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD}$] and [not $\hat{R}_{SSD} \trianglerighteq_{RA} \underline{R}_{SSD}$] and (b) *not comparable* if $\underline{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD} \trianglerighteq_{RA} \underline{R}_{SSD}$. Cases (a) and (b) are the two possible cases if [not $\underline{R}_{SSD} \triangleright_{RA} \hat{R}_{SSD}$] and [not $\hat{R}_{SSD} \triangleright_{RA} \underline{R}_{SSD}$].

Case (a) implies that the two investors have very similar preferences which do not, in the strict sense, disagree which each other. The two investors are, in a different sense, still comparable: The comparison leads to the conclusion that the preferences of the two investors are not sufficiently different. Indeed, we cannot reject the hypothesis that the two investors have the same preferences underlying their choices, and we can find rationalising utility functions which either imply that the first investor is more risk averse than the second or vice versa (see Theorem 3.3.iii). Case (b) implies that either (1) the extent of risk aversion of at least one of the investors is not constant over the entire income range, or (2) that the two investors have different notions of risk.

Figure 3.(a) gives an example of choices of two investors such that one is revealed more risk averse than the other. Assume for simplicity that $\pi = (\frac{1}{2}, \frac{1}{2})$. One of the investors chooses the riskless portfolio on both budgets, while the other exploits the unequal prices to choose more of the second asset, thus obtaining a higher expected value. We have $\underline{x}^1 \prec_E \hat{x}^1$ and $\underline{x}^1 \, \underline{P} \, \hat{x}^2$, and at the same time $\hat{x}^2 \, \hat{P} \, \underline{x}^1$, and as there are no observations to support $\hat{R}_{SSD} \trianglerighteq_{RA} \underline{R}_{SSD}$ we conclude that $\underline{R}_{SSD} \triangleright_{RA} \hat{R}_{SSD}$.

The example Figure 3.(b) shows the choices of two investors which lead to the conclusion that they are similar. Figure 3.(c) shows two sets of choices which are incomparable: Here, one investor, $\underline{R}$, is not enticed to take any risks at moderately steep budgets, but as the price ratio increases a bit more, he suddenly accepts risk in exchange for a high expected value. The other investor chooses a moderate risk in all four situations. In effect, we have $\underline{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD}$ based on the observations on the left, and $\hat{R}_{SSD} \triangleright_{RA} \underline{R}_{SSD}$ based on the observations on the right. See also Figure 8 in the appendix for four examples of recovered preferences from experimental choice data. For example, the subject shown in 8.(a) is revealed more risk averse than those in (b)-(d), while the subject shown in 8.(d) is incomparable to those in (b) and (c).

4 APPLICATION

4.1 Preliminaries

Theorem 2 provides a testable condition for SSD-rationalisation. If an investor does not satisfy SSD-GARP (or not even GARP), we would like to have a test for "almost optimising" behaviour, or a measure for the severity of the violation of the axiom. One such measure is the Afriat Efficiency Index (AEI, Afriat 1972) or Critical Cost Efficiency Index, which is arguably the most popular of such measures. Reporting the AEI is a standard in experimental economics.[5]

To obtain the AEI for GARP, budgets are shifted towards the origin until a set of observations satisfies GARP. We will use the same idea to measure efficiency of choices in terms of SSD-GARP: For $e \in [0,1]$, define the relations $R^0(e)$ and $P^0(e)$ as $x^i \, R^0(e) \, x^j$ if $e \, p^i x^i \geq p^i x$ and $x^i \, P^0(e) \, x^j$ if $e \, p^i x^i > p^i x$, and let $R(e) = (R^0(e))^+$ be the transitive closure. The relation $\succsim_{SSD}(e)$ is defined as $x \succsim_{SSD}(e) y$ if $e \, x \succsim_{SSD}(e) \, y$. Then define $R_{SSD}(e)$ and $P_{SSD}(e)$ accordingly as is Eq. (1). We then say that Ω satisfies SSD-GARP(e)

[5]See, for example, Sippel (1997), Mattei (2000), Harbaugh et al. (2001), Andreoni and Miller (2002), Février and Visser (2004), Choi et al. (2007b), Fisman et al. (2007).

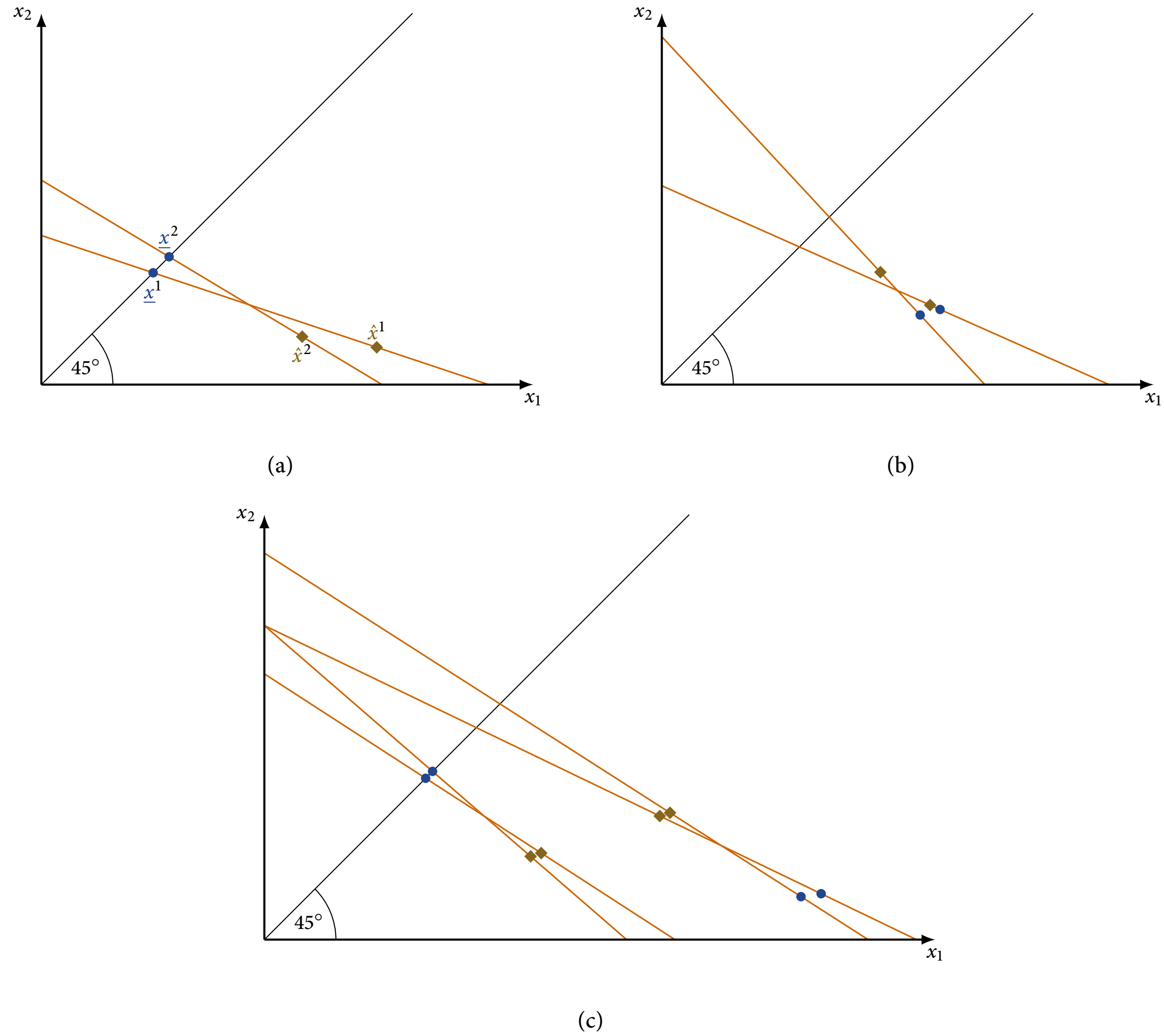


Figure 3: Example of the comparative approach based on revealed preference relations. All choices satisfy SSD-GARP for the probability vector $\pi = (½, ½)$. The choices by investor $\underline{R}$ are shown as ●, and the choices by investor $\hat{R}$ as ◆. In (a), $\underline{R}$ is revealed more risk averse than $\hat{R}$. In (b), the two investors have similar preferences. In (c), the two investors are incomparable.

if [not $x^i \, \mathrm{P}_{\mathrm{SSD}}(e) \, x^j$] whenever $x^j \, \mathrm{R}_{\mathrm{SSD}}(e) \, x^i$. Then the Ssd-Aei is the largest number e such that Ssd-Garp(e) is satisfied. The Aei, of course, is defined in the same way, applied to the R relation.

Note that the Aei can be interpreted as a measure of wasted income; that is, an investor with an Aei of, say, 0.9 could have obtained the same level of utility by spending only 90% of what he actually spent to obtain this level. Also note that this interpretation is practically the same when using the Ssd-Aei. Suppose a subject satisfies Garp, such that the Aei is 1, but the Ssd-Aei is .95. Then at least one choice x^i must be strictly preferred to a portfolio y which is preferred to x^i, that is, $e\, p^i x^i > p^i y$ for all $e > .95$. This portfolio y cannot be one of the choices, as that would already violate Garp. Thus, y must have Ssd over x^i, and the investor could have had at least the same utility from y as he got from x^i. But once we set e to .95, x^i is no longer strictly revealed preferred to y; thus we conclude that the investor wasted 5% of his income.

Bronars (1987) suggests a Monte Carlo approach to determine the power the test has against random behaviour. The approximate power of the test is the percentage of random choices which violate Garp; this can also be applies to Ssd-Garp. A high power does not, however, imply that the power remains high once we "allow" investors to deviate from 100% efficiency. This is also related to the problem that there is no natural definition for what constitutes a "high" or "low" Aei. But it is important to know what efficiency levels can be considered as high enough when screening the data for efficiency before further analytical steps are taken. Heufer (2012b) provides a detailed discussion of this point together with a procedure based on Monte-Carlo simulations and the reduction of the power the test has against random behaviour to determine which set of observations can be considered close enough to Garp. This can easily be adopted for Ssd-Garp. For the application to data in Section 4.2 we use the "measure of success" adaptation in Heufer (2012b) to determine which subjects to use. It is based on Selten's (1991) measure of predictive success for area theories and maximises the difference between the fraction of subjects and the fraction of random choice sets accepted as close enough to an axiom based on the efficiency index.

4.2 Data Analysis

We are using data by Choi et al. (2007a); for a detailed description the reader is referred to their article. Choi et al. asked ninety three subjects to choose one portfolio on each of fifty budget sets. In the symmetric treatment, the two assets paid off with probabilities $(\pi_1, \pi_2) = (½, ½)$. In the asymmetric treatment, the two assets paid off with probabilities $(\pi_1, \pi_2) = (⅓, ⅔)$. In one of the sessions the probabilities were $(\pi_1, \pi_2) = (⅔, ⅓)$ which is taken into account.

Hence, for each subject, we observe $N = 50$ portfolio choices on budgets of the form $B(p^i) = \{x \in \mathbb{R}^2_+ : p^i x = 1\}$ for $i = 1, \ldots, N$. There were $L = 2$ states, and the probabilities which with each state occurred were known by the subjects and remained fixed throughout the experiment. The only thing that changed between choices were the price vectors p^i (and, strictly speaking, wealth, which is here used to normalise prices), which were randomly drawn from a given distribution.

Sixteen of the subjects satisfies Garp, but even those subjects do not satisfy Ssd-Garp.[6] Like Choi et al. (2007a), we therefore compute efficiency indices for the subjects and for generated sets of random choices. Figures 4 and 5 show the distribution of the Ssd-Aei for subjects and random choices, for the two different treatments, based on 1860 random choice sets. While most subjects in the asymmetric treatment show substantially higher Ssd-efficiency than random choices, a notable fraction of 41.3% (17.39%) has

[6] Note that there are minor rounding errors in the data, which can lead to many more Garp violations if income is not adjusted. One of the Garp-consistent subjects has an Ssd-Aei of .7341, which is one of the lowest of all subjects in the asymmetric treatment. The choices indicate that this subject treated x_1 and x_2 as homogeneous goods despite the asymmetric probabilities. This highlights the importance of testing Ssd-Garp.

an efficiency level of less than .9 (.8), while this is the case for only 21.28% (12.77%) of subjects in the symmetric treatment. Subjects in the symmetric treatment have generally somewhat higher efficiency levels, but stochastic dominance is a rather simple concept with equal probabilities. It might indicate that a few subjects have some minor difficulties applying the concept of stochastic dominance in the asymmetric case.

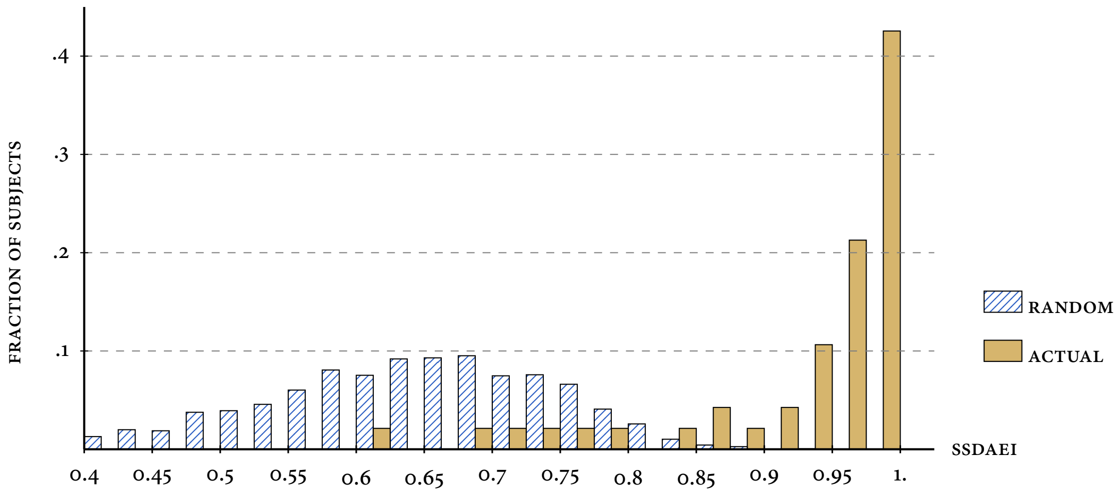


Figure 4: SSD-AEI for symmetric treatment: ▨ shows the distribution for random choices, ▬ for actual subjects. Data from Choi et al. (2007a).

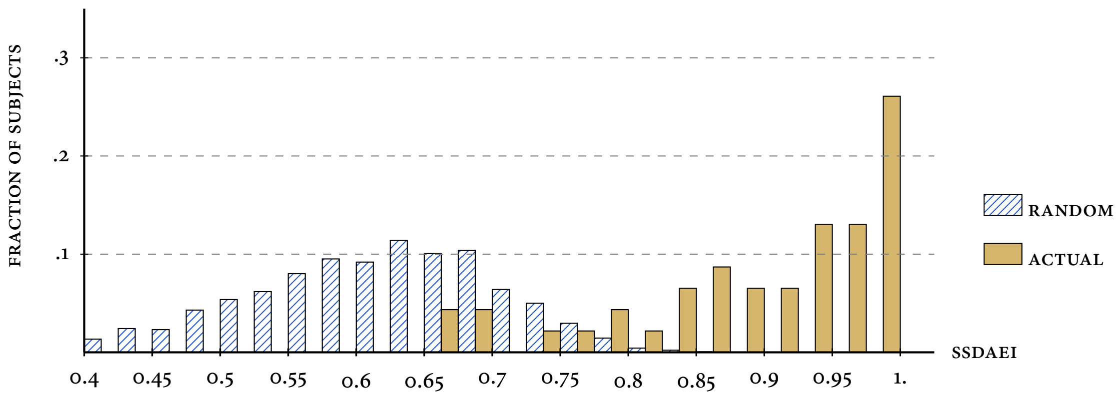


Figure 5: SSD-AEI for asymmetric treatment: ▨ shows the distribution for random choices, ▬ for actual subjects. Data from Choi et al. (2007a).

Tables 1 and 2 summarise some results. For the symmetric treatment, based on the procedures described in Heufer (2012b), we should consider an AEI and and SSD-AEI of $\bar{e} = .8401$ as sufficient. For the asymmetric treatment, these values are $\bar{e} = .8396$ for the AEI and $\bar{e} = .7791$ for the SSD-AEI. We require that subjects satisfy both requirements.

We compare the choices of subjects *corrected by their individual SSD-AEI-level*, that is, we base the comparison on the $R_{SSD}(e)$ relation, where e is the subject's SSD-AEI.[7] With 41 accepted subjects for the symmetric treatment (39 for the asymmetric treatment) we have 1640 (1482) comparisons. In 63.54% of

[7] We subtract an additional .001 from the efficiency level, as the computation of the efficiency levels is only an approximation.

all cases we find that one of the subject is revealed less or more risk averse than the other (54.25% for the asymmetric treatment). In 8.29% (12.96%) of the cases, neither subject is partially more risk averse than the other, that is, these subjects have similar preferences. In 28.17% (32.79%) of all cases, both subjects are partially revealed preferred to each other, rendering them incomparable.

We also compare subjects at the minimum Ssd-Aei-level of each pair of subjects, that is, we apply the same (low) efficiency standard to both of them, which somewhat increases the fraction of subjects who are comparable. Tables 1 and 2 summarise these main results.

SYMMETRIC TREATMENT			
	AEI	SSD-AEI	BOTH
EFFICIENCY REQUIREMENT $\bar{e}$	.8401	.8401	
NO. OF SUBJECTS WITH $e \geq \bar{e}$	41	41	41
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
SUBJECTS' AEI AND SSD-AEI	.9954		.9936
RANDOM AEI AND SSD-AEI	.9811		.9786
OF THOSE SUBJECTS WHICH SATISFY $\bar{e}$ REQUIREMENTS:			
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
AEI AND SSD-AEI	.9721		.9904
COMPARABILITY OF RISK AVERSION	MORE/LESS	NEITHER	BOTH
FRACTION AT INDIVIDUAL SSD-AEI	63.54%	8.29%	28.17%
FRACTION AT MINIMUM SSD-AEI	63.66%	20.73%	15.61%

Table 1: Summary statistics for the symmetric treatment with $(\pi_1, \pi_2) = (½, ½)$. See text for a description. Data from Choi et al. (2007a).

Choi et al. (2007a) estimate parameters α and ρ of a utility function $U : \mathbb{R}^L_+ \to \mathbb{R}$, where $U(x) = \min\{(\pi_2/\pi_1)\,\alpha\, u(x_1) + u(x_1), u(x_1) + (\pi_2/\pi_1)\,\alpha\, u(x_2)\}$ and $u : \mathbb{R}_+ \to \mathbb{R}$ takes the form of a power utility function $u(x_i) = x_i^{1-\rho}/(1-\rho)$. If $\alpha > 1$, this utility function exhibits disappointment aversion (Gul 1991). Thus, α is a measure of disappointment aversion, and ρ is the Arrow-Pratt measure of relative risk aversion.

We compare all subjects to choices generated by maximising the utility function U for different parameters. As parameters, we choose the α and ρ for different percentiles, that is, we use α and ρ such that 5%, 25%, 50%, 75%, and 95% of all subjects have the same or lower individual estimates. Table 3 shows the result for the symmetric treatment for which we find that the nonparametric comparison corresponds very well to the parameter estimates. For example, using the median α and ρ we find that at individual Ssd-Aei-levels 31.71% of subjects are less risk averse, 9.76% of subjects have similar preferences, and 34.15% of subjects are more risk averse. Table 4 shows the same result for the asymmetric treatment, where only 2.56% of subjects are less risk averse while 58.97% of subjects are more risk averse than the preferences described by a utility function with median parameters.

As Choi et al. (2007a) estimate a two-parameter utility function, they cannot represent risk aversion as a single parameter. They therefore compute a risk premium r for every subject, which is the fraction of initial wealth that gives the same utility as a lottery with 50-50 odds of winning or losing the initial amount. We can compare the ranking of subjects' risk aversion obtained by r with the nonparametric interpersonal comparison. If of two subjects, the first has a higher r than the second, then ideally the first subject is revealed more risk averse than the second. If this is not the case, and the second subject is revealed more

ASYMMETRIC TREATMENT			
	AEI	SSD-AEI	BOTH
EFFICIENCY REQUIREMENT $\bar{e}$	.8396	.7791	
NO. OF SUBJECTS WITH $e \geq \bar{e}$	42	40	39
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
SUBJECTS' AEI AND SSD-AEI	.7192		.6671
RANDOM AEI AND SSD-AEI	.8866		.8585
OF THOSE SUBJECTS WHICH SATISFY $\bar{e}$ REQUIREMENTS:			
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
AEI AND SSD-AEI	.6653		.6967
COMPARABILITY OF RISK AVERSION	MORE/LESS	NEITHER	BOTH
FRACTION AT INDIVIDUAL SSD-AEI	54.25%	12.96%	32.79%
FRACTION AT MINIMUM SSD-AEI	44.26%	35.63%	20.11%

Table 2: The same summary statistics as in Table 1, here for the asymmetric treatment with $(\pi_1, \pi_2) = (1/3, 2/3)$. Data from Choi et al. (2007a).

SYMMETRIC TREATMENT						
	CRRA		SUBJECT RISK AVERSION			
PERCENTILE	α	ρ	LESS	NEITHER	MORE	BOTH
5TH:	1.000	0.048	0.00%	0.00%	100.00%	0.00%
25TH:	1.000	0.165	0.00%	7.32%	87.80%	4.88%
50TH:	1.179	0.438	31.71%	9.76%	34.15%	24.39%
75TH:	1.477	0.794	68.29%	12.2%	4.88%	14.63%
95TH:	2.876	3.871	80.49%	9.76%	0.00%	9.76%

Table 3: Nonparametric comparison of subjects' risk aversion with a choices generated by a utility function with different parameters, here for the symmetric treatment. See text for a description. Data from Choi et al. (2007a).

ASYMMETRIC TREATMENT						
	CRRA		SUBJECT RISK AVERSION			
PERCENTILE	α	ρ	LESS	NEITHER	MORE	BOTH
5TH:	1.000	0.048	0.%	2.56%	92.31%	5.13%
25TH:	1.000	0.165	0.%	2.56%	82.05%	15.38%
50TH:	1.179	0.438	2.56%	17.95%	58.97%	20.51%
75TH:	1.477	0.794	41.03%	20.51%	17.95%	20.51%
95TH:	2.876	3.871	56.41%	0.00%	2.56%	41.03%

Table 4: The same statistics as in Table 3, here for the asymmetric treatment. Data from Choi et al. (2007a).

risk averse than the first or both have similar preferences, then the difference ranking of the two subjects by r should be small. Table 5 shows how often the ranking of two subjects, of which one is revealed more risk averse than the other, differ by more than 1, 2, 4, 8, and 12 ranks.

For the symmetric treatment, a measure of risk aversion can also be obtained by computing the share of tokens allocated to the cheaper asset. The higher the share, the less risk averse a subject should be. Table 5 also shows how often the ranking of two subjects differs by this measure of risk aversion, where we use the average share of tokens and call this measure $\tilde{r}$.

SYMMETRIC TREATMENT						
	BY MORE THAN . . . RANKS					
FRACTION OF COMPARISONS WHICH	0	1	2	4	8	12
DISAGREE WITH RANKING BY r	23.55%	20.04%	17.56%	14.26%	8.88%	6.20%
DISAGREE WITH RANKING BY $\tilde{r}$	23.84%	21.70%	19.18%	15.00%	9.47%	6.71%
ASYMMETRIC TREATMENT						
	BY MORE THAN . . . RANKS					
FRACTION OF COMPARISONS WHICH	0	1	2	4	8	12
DISAGREE WITH RANKING BY r	27.36%	25.05%	22.02%	17.33%	11.12%	7.94%

Table 5: Difference in ranking of subjects by measures of risk aversion and their nonparametric comparisons. See text for a description.

Tables 6 and 7 in the appendix give the complete list of interpersonal comparisons between all subjects in the symmetric and asymmetric treatment, respectively, at individual SSD-AEI-levels. Figure 8 in the appendix shows examples of revealed preferred and revealed worse sets of four different subjects based on the extended relation R_{SSD}. The first one is revealed more risk averse than most other subjects, the second on is revealed less risk averse than most other subjects. The third one is an intermediate case which is similar to several other subjects, and revealed more and revealed less risk averse to some others. The last one is a subject that is incomparable with several others. The last one is particularly interesting as it nicely illustrates why some subjects are not comparable: This subject exhibits almost risk neutrality around the 45° line, with a sudden sharp increase in risk aversion as the amount of any assets drops below 15.

5 DISCUSSION AND CONCLUSION

We have provided a method to account for first and second order stochastic dominance when analysing choice under uncertainty. This allows to test if there exists a well behaved utility function which rationalises such data and obeys stochastic dominance, and to extend the revealed preference relations recovered from such data. The application to the experimental data of Choi et al. (2007a) shows that while most subjects are reasonably close to such SSD-rationality, although some clearly are not. On the one hand, the result therefore confirms previously drawn conclusions to a large extent. On the other hand, it shows that there are, albeit few, subjects who come close to GARP but exhibit strong violations of SSD-rationality. This highlights that it is important to apply the tests for SSD.

This analysis enabled us to provide a way to make Yaari's (1969) idea for comparative risk aversion operational based on revealed preferred and revealed worse sets. The central rationalisability theorem shows that if and only if the conditions for "revealed more risk averse" are satisfied, there exist utility functions which rationalise the two observations on two investors, such that the utility function of the more risk averse investor exhibits greater risk aversion for every portfolio. Furthermore there do not exist rationalising utility functions which exhibit greater risk aversion for the less risk averse investor.

The theorem also shows that it is sufficient to only compare a finite number of portfolios, namely those observed as choices, even though the revealed more risk averse relation is defined in terms of the revealed preferred and worse sets of all portfolios. It therefore leads to a nonparametric way to compare the risk aversion of two investors without relying on particular forms of utility.

Testing the experimental data of Choi et al. (2007a) for consistency with SSD-rationality shows that, compared to random choices, strong consistency of most subjects is confirmed. The nonparametric approach to comparative risk aversion is useful as an alternative or complement to parametric estimation of risk aversion. It can serve as a robustness check for the parametric approach; the analysis in Choi et al. (2007a) is found to be quite robust for both treatments, but more so for the symmetric treatment. Obviously a nonparametric approach does not offer a distribution of parameters to describe risk attitudes in a given sample. However, it can be used to compute the fraction of investors which are less or more risk averse than *any* given preference and can therefore also offer a characterisation of risk preferences in a population.

Interpersonal comparisons based on revealed preferred and worse sets can also be usefully applied to other aspects of preferences, such as sense of fairness (Karni and Safra 2002a,b) or impartiality (Nguema 2003). For example, Karni and Safra (2002b) apply Yaari's (1969) notion of "is more risk averse than" to the concept of "has a stronger sense of fairness than". The results here can be translated to suit this interpersonal comparison of the sense of fairness. A first step towards such an analysis has been recently made by Heufer (2012a).

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A APPENDIX

A.1 *Arrow-Debreu Securities and General Assets*

The basic arbitrage financial principle has been described by Ross (1978). A clear exposition of the facts we use here can be found in Varian (1987), based on the work of Breeden and Litzenberger (1978) who derive "state prices" from option pricing, among others. We will only provide some intuitions for the claim that the portfolio choice problem with Arrow-Debreu securities can be obtained by a transformation of a (superficially) more general problem. That the examples here generalise follows from previous work (see Varian 1987).

There are $K \geq L$ linearly independent general assets $Y_{\cdot,i}$, $i = 1, \ldots, K$. Asset $Y_{\cdot,i}$ pays off $Y_{j,i} \geq 0$ in state j, $j = 1, \ldots, L$. Let Y be an $L \times K$ matrix, where column i represents state contingent payoffs of asset i and row j represents payoffs of assets in state j. Without loss of generality, let the price of each asset $Y_{\cdot,i}$ be 1.[8] An investor who invests an amount $w > 0$ chooses a portfolio $y = (y_1, \ldots, y_K)'$ such that $\sum_{i=1}^{K} y_i = w$. Then the payoff of this portfolio in state j is $Y_{j,\cdot}y$. Without short-selling, any state contingent payoff vector that can be obtained by the investor is a linear combination of the assets $Y_{\cdot,i}$ which is an affordable portfolio, that is, is given by $z = (z_1, \ldots, z_L) = \left(\sum_{i=1}^{K} y_i Y_{1,i}, \ldots, \sum_{i=1}^{K} y_i Y_{L,i}\right)$, such that $y_i \geq 0$ for $i = 1, \ldots, K$, and $\sum_{i=1}^{K} y_i = w$. If short-selling is allowed (i.e. if the investor can "buy" a negative amount of some assets), the condition $y_i \geq 0$ is dropped. In any case, we require that the state contingent payoff vector has no negative entries ($z_j \geq 0$ for all $j = 1, \ldots, L$), that is, we do not allow the investors to risk bankruptcy.

The "no arbitrage" condition required in this context states that it is not possible to construct a portfolio consisting of short- and long-positions (negative and positive amounts of assets) such that $\sum_{i=1}^{K} y_i = 0$, $z_j \geq 0$ for all $j = 1, \ldots, L$ and $z_j > 0$ for at least one j. As prices of assets are fixed, this condition states that there does not exist an asset which is dominated by a linear combination of other assets. Suppose $L = 2$ and $K = 3$, and $Y_{\cdot,1} = (6, 2)$, $Y_{\cdot,2} = (2, 6)$ are given. These two assets are contained in a hyperplane in $\mathbb{R}^2$. Then $Y_{\cdot,3}$ must be contained in the same hyperplane: Suppose that this is not the case because $Y_{\cdot,3} = (3, 3)$, i.e. the third asset is dominated by, for example, one half of each of the first and second asset. Then an investor can short-sell two units of $Y_{\cdot,3}$ and use the revenue to buy one unit of $Y_{\cdot,1}$ and $Y_{\cdot,2}$ each. Then $z = (2, 2)$ and he obtains a payoff of 2 with certainty even though $\sum_{i=1}^{K} y_i = 0$. See Figure 6(a).

If the no arbitrage condition holds, then all available assets are contained in an (L-1)-dimensional hyperplane in $\mathbb{R}^L$. Then any state contingent payoff vector that can be obtained by spending w on a portfolio is a point in $\mathbb{R}^L_+$ on or below that hyperplane, and the hyperplane itself is the efficiency frontier. But then the set of efficient state contingent payoff vectors is the convex hull of the intersections of that hyperplane with the standard basis vectors of $\mathbb{R}^L$. Suppose for example that $L = K = 2$, and $Y_{\cdot,1} = (3, 2)$, $Y_{\cdot,2} = (1, 6)$, and $w = 1$. Then $(4, 0)$ and $(0, 8)$ are intersections of the hyperplane containing the two assets with the axes (see Figure 6(b)), and the set of all efficient z is given by the convex hull of these two points (as we do not allow the risk of bankruptcy). Thus the choice situation can be transformed into an equivalent situation where the investor spends one unit of money on two Arrow-Debreu securities $X_{\cdot,1} = (1, 0)$ and $X_{\cdot,2} = (0, 1)$ with prices 1/4 and 1/8, respectively.

That this generalises follows from results known in the literature (e.g. Varian 1987). In the previous example, the choice situation with general assets can be transformed into the framework of this paper by specifying a budget with bounding hyperplane $\{x \in \mathbb{R}^L_+ : (1/4)x_1 + (1/8)x_2 = 1\}$ if short-selling is allowed.

[8] One can think about this as a normalisation of asset payoffs, that is, state contingent payoffs specify what an investor gets for investing one unit of money in the asset.

If short-selling is not allowed, the budget set is a subset of B, in particular, the efficient frontier of the budget is $\{x \in \mathbb{R}^L_+ : (1/4)x_1 + (1/8)x_2 = 1 \text{ and } x_1 \leq 6 \text{ and } x_2 \leq 12\}$. See Figure 6.(c) and 6.(d) for illustrations.

As already elaborated on in Section 3, the riskless portfolio of a budget (i.e., the portfolio which pays off the same amount in every state) can be considered the status quo of the budget situation. Depending on the budget and preferences, the investor either has the opportunity to improve upon the status quo, or the status quo is optimal compared to the other alternatives in the budget. If a utility maximising investor does not choose the riskless portfolio, then he reveals a part of his acceptance set in terms of Yaari (1969). Clearly, a risk averse investor will not choose a portfolio which has a lower expected value than the status quo, as this portfolio cannot offer more certainty than the status quo and is stochastically dominated by the status quo. Figure 7 illustrates this for $L = 2$ and the probability vector $\pi = (½, ½)$. The riskless portfolio y of the budget is on the 45° line. The set of portfolios with the same expected value as the riskless portfolio is indicated by the dashed line. As y is riskless, we have that $y \succsim_{\text{SSD}} z$ whenever $y \sim_{\text{E}} z$. Thus, all portfolios in the budget with a lower expected value than y are stochastically dominated by y and will not be preferred over y by any risk averse utility maximiser. However, some of the available alternatives with a higher expected value than y might be preferred over y by an investor who is not too risk averse.

The indicated choice x^1 of an investor reveals that this investor prefers x^1 over the riskless portfolio. In terms of Yaari (1969), x^1 is in the acceptance set of the status quo y. Then by convexity of preferences (or concavity of the utility function), every convex combination of x^1 and y must at least be weakly preferred to y. Thus, the line segment connecting x^1 and y must be part of the investors acceptance set.

A.2 *Proof of the Lemmata*

We only consider the SSD case here; proofs for the FSD case are simpler.

Proof of Lemma 1 This follows directly from the fact that every risk averse expected utility maximiser will prefer x over y whenever $x \succsim_{\text{SSD}} y$: Let $\mathbf{EU}_u(x)$ denote the expected utility of $x \in \mathbb{R}^L_+$ with $u : \mathbb{R}^L_+ \to \mathbb{R}$ being a continuous, increasing, and concave utility function. Then $x \succsim_{\text{SSD}} y$ if and only if $\mathbf{EU}_u(x) \geq \mathbf{EU}_u(y)$ for all such u. Suppose $z = \mu x + (1-\mu)y$ for $\mu \in (0,1)$; then $\mathbf{EU}_u(x) \geq \mathbf{EU}_u(y)$ implies $\mathbf{EU}_u(z) \geq \mathbf{EU}_u(y)$, and thus $z \succsim_{\text{SSD}} y$. ■

Proof of Lemma 2

(i) Let $ma(x,i)$ denote the maximal value of y_i such that $y \succsim_{\text{SSD}} x$. Then the set

$$HC(x) = \{y \in \mathbb{R}^L_+ : \min(\{y_1, \ldots, y_L\}) \geq \min(\{x_1, \ldots, x_L\}) \\ \text{and } y_i \leq ma(x,i) \text{ for all } i = 1, \ldots, L\}$$

is a hypercube in $\mathbb{R}^L_+$ which intersects the hyperplane $\mathcal{P}(x, \sim_{\text{E}})$ (except when $x_i = x_j$ for all $i, j = 1, \ldots, L$, in which case the two sets only share the point x). Then $\mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}}) \subseteq HC(x) \cap \mathcal{P}(x, \sim_{\text{E}})$. By construction of $\hat{M}(x)$, $HC(x) \cap \mathcal{P}(x, \sim_{\text{E}}) = \text{CH}(\hat{M}(x))$ and $y \succsim_{\text{SSD}} x$ for all $y \in \hat{M}(x)$. Then by Lemma 1, $\text{CH}(\hat{M}(x)) \subseteq \mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$, and the first part of Lemma 2 follows.

(ii) It is obvious that $\mathcal{P}(x, \succsim_{\text{SSD}}) \subseteq \overline{\text{CMH}}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$. As $y \succsim_{\text{E}} x$ is a necessary condition for $y \succsim_{\text{SSD}} x$, consider any $y \succ_{\text{E}} x$, $y \notin \overline{\text{CMH}}(\hat{M}(x))$, and suppose $y \succsim_{\text{SSD}} x$. Let $y_j = \max(y)$ and let $z \sim_{\text{E}} x$ be such that $z_i = y_i$ for all $i \neq j$ and $z_j < y_j$. Then $F(\xi^i, y) = F(\xi^i, z)$ for all $\ell < n$. Thus if $y \succsim_{\text{SSD}} x$ then $z \succsim_{\text{SSD}} x$. But that contradicts the first part of Lemma 2.

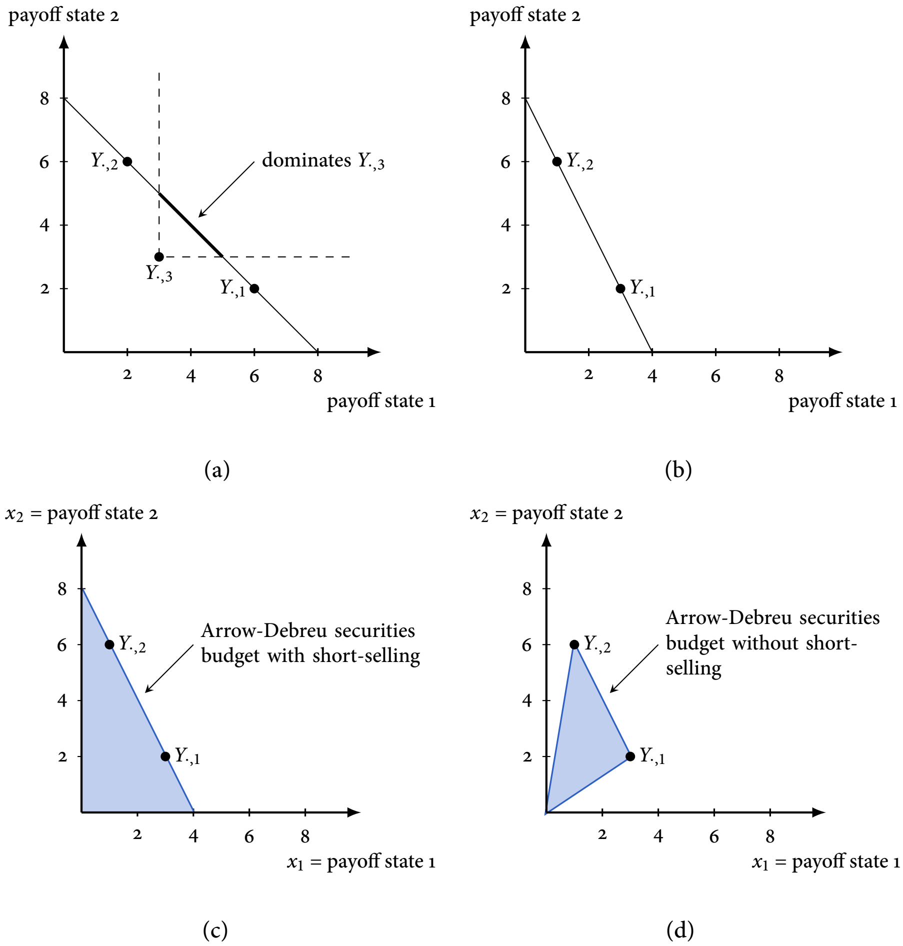

Figure 6: (a): Example with three general assets where the no arbitrage condition does not hold. (b): Example with two assets and no arbitrage. (c): Possible state-contingent payoffs achievable with $w = 1$ and budget for Arrow-Debreu securities for the example in (b). (d): Same as in (c), but without short-selling.

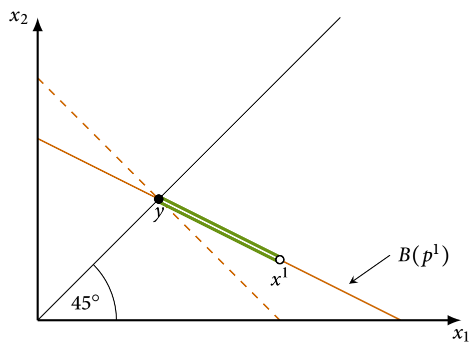

Figure 7: Example with probability vector $\pi = (\frac{1}{2}, \frac{1}{2})$. The portfolio y is riskless as it pays off the same amount in both states. The dashed line shows the set of portfolios with the same expected value as y. A risk averse utility maximiser will only choose y or alternatives to the right of y, depending on his degree of risk aversion. If y is interpreted as a status quo, then by convexity of preferences, the line segment connecting y and x^1 must be part of the investors acceptance set, that is, the gambles he is willing to accept.

■

A.3 Proof of Theorem 2

Again, we only proof the SSD case here. Let $\mathcal{U}_{\text{SSD}} \subset \mathcal{U}$ be the set of all utility functions which are consistent with the $\succsim_{\text{SSD}}$ ordering, that is, if $u \in \mathcal{U}_{\text{SSD}}$, then $u(x) \geq u(y)$ whenever $x \succsim_{\text{SSD}} y$. Note that $\mathcal{U}_{\text{SSD}} \neq \varnothing$ (see also the proof of Lemma 1).

Lemma 3 *If $u \in \mathcal{U}$ SSD-rationalises Ω, then $u \in \mathcal{U}_{\text{SSD}}$. If $u \in \mathcal{U}_{\text{SSD}}$ rationalises Ω, then u also SSD-rationalises Ω.*

Proof Recall the definition of R_{SSD} as $(\text{R} \cup \succsim_{\text{SSD}})^{+}$. Thus, $\succsim_{\text{SSD}} \subseteq \text{R}_{\text{SSD}}$ and the first statement follows. For the second statement, suppose that $u \in \mathcal{U}_{\text{SSD}}$ rationalises Ω. Then $u(x) \geq u(y)$ whenever $x \,\text{R}\, y$, and because $u \in \mathcal{U}_{\text{SSD}}$, we also have $u(x) \geq u(y)$ whenever $x \succsim_{\text{SSD}} y$. Then by transitivity of R, $\succsim_{\text{SSD}}$, and R_{SSD}, the result follows. ■

Proof of Theorem 2
(1) ⇒ (2): The proof is very similar to the proof of Afriat's Theorem that can be found in Varian (1982) and we omit it.
(2) ⇒ (1): The existence of a utility function $u \in \mathcal{U}$ which rationalises Ω follows from Theorem 1 because SSD-GARP implies GARP. Suppose that Ω satisfies SSD-GARP (and therefore GARP) but there does not exist a utility function which SSD-rationalises Ω. Then by Lemma 3, there does not exist a $u \in \mathcal{U}_{\text{SSD}}$ which rationalises Ω. Then *for all $u \in \mathcal{U}_{\text{SSD}}$*, it must be the case that for some set of portfolios $y^1, y^2, y^3, \ldots$, either

(i) $[u(x) < u(y^1)$ or $u(x) < u(y^2)$ or $u(x) < u(y^3)$ or $\ldots]$ and $[x \,\text{R}\, y^i$ for some x and for all $i = 1, 2, 3, \ldots]$; or
(ii) $[u(x) \leq u(y^1)$ or $u(x) \leq u(y^2)$ or $u(x) \leq u(y^3)$ or $\ldots]$ and $[x \,\text{P}\, y^i$ for some x and for all $i = 1, 2, 3, \ldots]$; or
(iii) $[u(y^1) < u(x)$ or $u(y^2) < u(x)$ or $u(y^3) < u(x)$ or $\ldots]$ and $[y^i \,\text{R}\, x$ for some x and for all $i = 1, 2, 3, \ldots]$; or
(iv) $[u(y^1) \leq u(x)$ or $u(y^2) \leq u(x)$ or $u(y^3) \leq u(x)$ or $\ldots]$ and $[y^i \,\text{P}\, x$ for some x and for all $i = 1, 2, 3, \ldots]$.

Otherwise, some $u \in \mathcal{U}_{\text{SSD}}$ rationalises Ω, and by Lemma 3 SSD-rationalises Ω.

Note that by definition $x \,\text{R}\, y$ implies that $x \in \{x^i\}_{i=1}^N$, that is, x is one of the portfolios observed as choices.

In case (i), suppose that $x = x^i$ and $x^i \,\text{R}^0\, y^j$ for all $j = 1, 2, 3, \ldots$, and $u(x^i) < u(y^j)$ for at least one j. Then $y^j \succ_{\text{SSD}} x^i$, and therefore $y^j \in \mathcal{P}(x^i, \succ_{\text{SSD}})$ and by Lemma 2, $\mathcal{P}(x^i, \succsim_{\text{SSD}}) = \overline{\text{CMH}}(\hat{M}(x^i))$, thus $y \in \text{CMH}(\hat{M}(x^i))$. But $y^j \in B^i$, and $B(p^i)$ is a hyperplane which separates $\mathbb{R}^L_+$ into two half-spaces. Then as $y^j \in \text{CMH}(\hat{M}(x^i))$, at least one vertex of $\overline{\text{CMH}}(\hat{M}(x^i))$ must be in $\text{int}B^i$. By construction, the vertices of $\overline{\text{CMH}}(\hat{M}(x^i))$ consist only of a subset of $\tau(\Omega)$. Thus, there is at least one $t^k \in \tau(\Omega)$ such that $t^k \in \text{int}B^i$, and therefore $x^i \,\text{P}^0\, t^k$. But $t^k \succsim_{\text{SSD}} x^i$ and therefore $t^k \,\text{R}_{\text{SSD}}\, x^i$ which violates SSD-GARP, a contradiction.

Suppose instead that $x^i \,\text{R}\, x^j \,\text{R}^0\, y^j$. Then by the same arguments as in the preceding paragraph we find that $x^j \,\text{P}^0\, t^k$ and $t^k \succsim_{\text{SSD}} x^i$. But then $t^k \,\text{R}_{\text{SSD}}\, x^i$ and $x^i \,\text{P}^0\, t^k$ which violates SSD-GARP, a contradiction.

Case (ii) is quite similar, except that $y^j \in \overline{\text{CMH}}(\hat{M}(x^i))$. Then if $x^i \,\text{P}^0\, y$, $t^k \in \text{int}B^i$ follows because $y \in \text{int}B^i$, and similarly for $x^i \,\text{P}\, x^j \,\text{R}^0\, y^j$ and $x^i \,\text{R}\, x^j \,\text{P}^0\, y^j$.

In case (iii), as $y^i \mathrel{\text{R}} x$, we must have $y^i = x^i$ for some observed portfolio choice x^i. Then $x \succ_{\text{SSD}} x^i$ and $x^i \mathrel{\text{R}} x$. That this violates Ssd-Garp follows from the same arguments as in case (i).

In case (iv), we have $x \succsim_{\text{SSD}} x^i$ and $x^i \mathrel{\text{P}} x$. That this violates Ssd-Garp follows from the same arguments as in case (iv). ∎

A.4 Proof of Theorem 2

Lemma 4 *Suppose $\underline{\Omega}$ and $\hat{\Omega}$ satisfy Ssd-Garp. Then there exist choices of the two investors, $\underline{x}^j$ and $\hat{x}^i$, such that $[\hat{x}^i \mathrel{\hat{\text{R}}_{\text{SSD}}} \underline{x}^j$ and $\underline{x}^j \mathrel{\underline{\text{P}}_{\text{SSD}}} \hat{x}^i]$ or $[\underline{x}^i \mathrel{\underline{\text{R}}_{\text{SSD}}} \hat{x}^j$ and $\hat{x}^j \mathrel{\hat{\text{P}}} \underline{x}^i]$ if and only if $\mathcal{RP}(x^0, \underline{\text{R}}_{\text{SSD}}) \cap \mathcal{RW}(x^0, \hat{\text{R}}_{\text{SSD}}) \neq \varnothing$.*

Proof We will first show that the Lemma holds for $\underline{\text{R}}$ and $\hat{\text{R}}$ instead of $\underline{\text{R}}_{\text{SSD}}$ and $\hat{\text{R}}_{\text{SSD}}$. By Garp there is no $x \in \mathcal{RP}(x^0, \hat{\text{R}})$ such that $x^0 \geq x$. Then by the definition of $\mathcal{RW}(\cdot, \underline{\text{R}})$, for all $x \in \mathcal{RW}(x^0, \underline{\text{R}})$, $\underline{p}^i \underline{x}^i \geq \underline{p}^i x \Leftrightarrow x \in B(\underline{p}^i)$ for at least one $i = 1, \ldots, \underline{N}$. As $B(\underline{p}^i)$ is a hyperplane and, by Proposition 1, $\mathcal{RP}(x^0, \hat{\text{R}})$ is a convex polytope whose vertices are x^0 and all $\hat{x}^j \mathrel{\hat{\text{R}}} x^0$, there is at least one $\hat{x}^j \in \mathcal{RP}(x^0, \hat{\text{R}}) \cap \mathcal{RW}(x^0, \underline{\text{R}})$. By definition, $\hat{x}^j \in \mathcal{RW}(x^0, \underline{\text{R}})$ implies that $\hat{x}^j$, if chosen by consumer $\underline{\text{R}}$, cannot be revealed preferred to x^0 without violating Garp: If $\hat{x}^j \mathrel{\underline{\text{R}}} x^0$, then $\hat{x}^j \mathrel{\underline{\text{R}}} \underline{x}^k$ and $\underline{x}^k \mathrel{\underline{\text{P}}} \hat{x}^j$. But $\hat{x}^j \mathrel{\hat{\text{R}}} x^0$, thus $\hat{x}^j \mathrel{\hat{\text{R}}} \underline{x}^k$. Then $\hat{x}^j \mathrel{\hat{\text{R}}} \underline{x}^k$ and $\underline{x}^k \mathrel{\underline{\text{P}}} \hat{x}^j$; and similarly for $[\underline{x}^i \mathrel{\underline{\text{R}}} \hat{x}^j$ and $\hat{x}^j \mathrel{\hat{\text{P}}} \underline{x}^i]$. Thus the Lemma holds for $\underline{\text{R}}$ and $\hat{\text{R}}$. That it holds for $\underline{\text{R}}_{\text{SSD}}$ and $\hat{\text{R}}_{\text{SSD}}$ (as stated) follows from the fact that $\succsim_{\text{SSD}}$ is the same for both investors. ∎

Lemma 5 *Suppose $\underline{\Omega}$ and $\hat{\Omega}$ satisfy Ssd-Garp. Then $\underline{\text{R}}_{\text{SSD}} \trianglerighteq_{\text{RA}} \hat{\text{R}}_{\text{SSD}}$ if and only if $\delta(\underline{\Omega}, \hat{\Omega}) = 1$.*

Proof The theorem states that $\underline{\text{R}}_{\text{SSD}} \trianglerighteq_{\text{RA}} \hat{\text{R}}_{\text{SSD}} \Leftrightarrow \delta(\underline{\Omega}, \hat{\Omega}) = 1$. It is obvious that $\delta(\underline{\Omega}, \hat{\Omega}) = 1 \Rightarrow \underline{\text{R}}_{\text{SSD}} \trianglerighteq_{\text{RA}} \hat{\text{R}}_{\text{SSD}}$. We will show that $\delta(\underline{\Omega}, \hat{\Omega}) = 0$ implies [not $\underline{\text{R}}_{\text{SSD}} \trianglerighteq_{\text{RA}} \hat{\text{R}}_{\text{SSD}}$].

Suppose $\delta(\underline{\Omega}, \hat{\Omega}) = 0$ and $\underline{\text{R}}_{\text{SSD}} \trianglerighteq_{\text{RA}} \hat{\text{R}}_{\text{SSD}}$. Then there does not exist a $\underline{x}^i \precsim_{\text{E}} \hat{x}^j$ such that $\underline{x}^i \mathrel{\underline{\text{R}}_{\text{SSD}}} \hat{x}^j \mathrel{\hat{\text{P}}_{\text{SSD}}} \underline{x}^i$, but still $\mathcal{RPL}(z^0, \underline{\text{R}}_{\text{SSD}}) \cap \mathcal{RWL}(z^0, \hat{\text{R}}_{\text{SSD}}) \neq \varnothing$. Then by Proposition 1 and Lemma 4, there is an $\underline{t}^i \in \tau(\underline{\Omega})$ such that $\underline{t}^i \in \mathcal{RPL}(z^0, \underline{\text{R}}_{\text{SSD}}) \cap \mathcal{RWL}(z^0, \hat{\text{R}}_{\text{SSD}})$. By Ssd-Garp and Theorem 2, we cannot have $z^0 \succ_{\text{SSD}} \underline{t}^i$, and because $z^0 \succ_{\text{E}} \underline{t}^i$, we cannot have $\underline{t}^i \succsim_{\text{SSD}} z^0$. Then either $\underline{t}^i = \underline{x}^i \mathrel{\underline{\text{R}}} z^0$ or there is an $\underline{x}^i$ such that $\underline{t}^i \succsim_{\text{SSD}} \underline{x}^i \mathrel{\text{R}} z^0$; in either case, $\underline{x}^i \in \mathcal{RPL}(z^0, \underline{\text{R}}_{\text{SSD}}) \cap \mathcal{RWL}(z^0, \hat{\text{R}}_{\text{SSD}})$.

As $\underline{x}^i \in \mathcal{RWL}(z^0, \hat{\text{R}}_{\text{SSD}})$ and [not $z^0 \succsim_{\text{SSD}} \underline{t}^i$], there must be some $\hat{t}^j \mathrel{\hat{\text{R}}_{\text{SSD}}} \underline{x}^i$, such that either (i) $z^0 \mathrel{\hat{\text{R}}_{\text{SSD}}} \hat{t}^j$ or (ii) $z^0 \succsim_{\text{SSD}} \mu\, \hat{t}^j + (1-\mu)\underline{x}^i$ for some $\mu \in (0,1)$. In case (ii), $\hat{t}^j = \hat{x}^j$, $\hat{x}^j \succ_{\text{E}} z^0$, and $\hat{x}^j \mathrel{\hat{\text{R}}} \underline{x}^i$; but then $\delta(\underline{\Omega}, \hat{\Omega}) = 1$, a contradiction. Thus, $z^0 \mathrel{\hat{\text{R}}_{\text{SSD}}} \hat{t}^j$. Because $\hat{t}^j = \hat{x}^j = z^0$ implies $\delta(\underline{\Omega}, \hat{\Omega}) = 1$, $z^0 \mathrel{\hat{\text{R}}_{\text{SSD}}} \hat{t}^j$ implies $z^0 \succsim_{\text{SSD}} \hat{t}^j$ as z^0 cannot be preferred to $\hat{t}^j$ in any other way.

Then $\underline{x}^i \mathrel{\underline{\text{R}}_{\text{SSD}}} z^0$ and $z^0 \succsim_{\text{SSD}} \hat{t}^j$ imply $\underline{x}^i \mathrel{\underline{\text{R}}_{\text{SSD}}} \hat{t}^j \succsim_{\text{SSD}} \hat{x}^j$, where $\hat{t}^j \in \hat{M}(\hat{x}^j)$. But then $\underline{x}^i \mathrel{\underline{\text{R}}_{\text{SSD}}} \hat{x}^j$, thus $\delta(\underline{\Omega}, \hat{\Omega}) = 1$ implies that $\underline{x}^i \succ_{\text{E}} \hat{x}^j$. Then $\hat{t}^j \sim_{\text{E}} \hat{x}^j$ implies [not $\hat{t}^j \succsim_{\text{SSD}} \underline{x}^i$], thus $\hat{x}^j \mathrel{\hat{\text{R}}_{\text{SSD}}} \underline{x}^j$.

To summarise, we have $z^0 \succsim_{\text{SSD}} \underline{x}^i$, $\underline{x}^i \succ_{\text{E}} \hat{x}^j$, $\underline{x}^i \mathrel{\underline{\text{R}}_{\text{SSD}}} \hat{x}^j$, and $\hat{x}^j \mathrel{\hat{\text{R}}_{\text{SSD}}} \underline{x}^i$. Then with

$$\mathcal{P}(z^0, \precsim_{\text{SSD}}) \cap \mathcal{P}(\underline{x}^i, \prec_{\text{E}}) \subseteq \mathcal{P}(\underline{x}^i, \prec_{\text{SSD}}),$$

we obtain that $z^0 \succsim_{\text{SSD}} \hat{x}^j$ and $\underline{x}^i \succ_{\text{E}} \hat{x}^j$ implies $\underline{x}^i \succ_{\text{SSD}} \hat{x}^j$. But $\hat{x}^j \mathrel{\hat{\text{R}}_{\text{SSD}}} \underline{x}^i$, which contradicts Ssd-Garp. ∎

Proof of Theorem 3 By assumption, the data satisfy Ssd-Garp, thus Ssd-rationalising utility functions exist.

The equivalence of (i) and (ii) for all three parts of the theorem follows immediately from Lemma 5.

By definition of the revealed preferred and worse sets and rationalisation of a utility function, if $\mathcal{RPL}(y, \underline{R}_{SSD}) \cap \mathcal{RWL}(y, \hat{R}_{SSD}) = x$, then for all $\underline{u}$ and $\hat{u}$ which Ssd-rationalise $\underline{\Omega}$ and $\hat{\Omega}$, respectively, $\underline{u}(x) > \underline{u}(y)$ and $\hat{u}(x) < \hat{u}(y)$. Conversely, if for some $x \prec_E y$, all $\underline{u}$ and $\hat{u}$ which Ssd-rationalise $\underline{\Omega}$ and $\hat{\Omega}$ must be such that $\underline{u}(x) > \underline{u}(y)$ and $\hat{u}(x) < \hat{u}(y)$, then $x \in \mathcal{RPL}(y, \underline{R}_{SSD})$ and $x \in \mathcal{RWL}(y, \hat{R}_{SSD})$. Thus, (ii) ⇔ (iii) for all three parts of the Theorem. ■

A.5 *Tables: Interpersonal Comparisons of Subjects*

Tables 6 and 7 show the complete list of interpersonal comparisons between all subjects in the symmetric and asymmetric treatment, respectively.

A.6 *Figure: Examples of Revealed Preferred and Worse Sets*

Figure 8 shows examples of revealed preferred and worse sets for some subjects. (a) Subject number 23 (ID 304): A subject who is revealed more risk averse than most other subjects. (b) Subject number 26 (ID 307): A subject who is revealed less risk averse than most other subjects. (c) Subject number 8 (ID 208): A subject who is revealed more risk averse and revealed less risk averse than some other subject and has similar preferences as many other subjects. (d) Subject number 5 (ID 205): A subject who is incomparable with some other subjects. Data from Choi et al. (2007a).

Symmetric Treatment: Part I																					
	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	19	20	21	22	23
2	•	¤	–	–	▶	–	◁	◁	▶	¤	◁	–	–	–	–	–	◁	▶	◁	▶	▶
3	¤	•	–	–	–	◁	▶	–	–	–	◁	▶	–	◁	◁	–	◁	¤	◁	¤	–
4	–	–	•	–	–	◁	–	–	▶	◁	◁	–	–	◁	–	–	◁	–	◁	¤	▶
5	–	–	–	•	▶	–	–	▶	▶	▶	◁	▶	–	–	–	¤	–	▶	–	–	▶
6	◁	–	–	◁	•	◁	◁	◁	–	▶	◁	–	–	◁	◁	◁	–	–	◁	–	–
7	–	▶	▶	–	▶	•	▶	◁	▶	▶	▶	–	–	▶	–	–	▶	▶	◁	▶	▶
8	▶	◁	–	–	▶	◁	•	◁	▶	¤	◁	▶	–	¤	▶	–	¤	▶	◁	¤	▶
9	▶	–	–	◁	▶	▶	▶	•	▶	▶	¤	–	–	▶	▶	–	▶	▶	◁	▶	–
10	◁	–	◁	◁	–	◁	◁	◁	•	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	¤
12	¤	–	▶	◁	◁	◁	¤	◁	▶	•	◁	–	–	◁	▶	–	◁	▶	◁	◁	▶
13	▶	▶	▶	▶	▶	◁	▶	¤	▶	▶	•	▶	¤	¤	▶	▶	▶	▶	◁	▶	▶
14	–	◁	–	◁	–	–	◁	–	▶	–	◁	•	–	◁	¤	◁	◁	–	◁	▶	▶
15	–	–	–	–	–	–	–	–	▶	–	¤	–	•	–	–	–	▶	–	–	–	▶
16	–	▶	▶	–	▶	◁	¤	◁	▶	▶	¤	▶	–	•	▶	▶	▶	▶	◁	▶	▶
17	–	▶	–	–	▶	–	◁	◁	▶	◁	◁	¤	–	◁	•	–	◁	▶	◁	¤	▶
18	–	–	–	¤	▶	–	–	–	▶	–	◁	▶	–	◁	–	•	–	–	–	–	▶
19	▶	▶	▶	–	–	◁	¤	◁	▶	▶	◁	▶	◁	◁	▶	–	•	–	◁	▶	▶
20	◁	¤	–	◁	–	◁	◁	◁	▶	◁	◁	–	–	◁	◁	–	–	•	◁	◁	▶
21	▶	▶	▶	–	▶	▶	▶	▶	▶	▶	▶	▶	–	▶	▶	–	▶	▶	•	▶	▶
22	◁	¤	¤	–	–	◁	¤	◁	▶	▶	◁	◁	–	◁	¤	–	◁	▶	◁	•	▶
23	◁	–	◁	◁	–	◁	◁	–	¤	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	•
24	–	–	–	–	–	◁	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
25	◁	◁	–	◁	◁	◁	◁	◁	▶	◁	◁	–	–	◁	–	◁	◁	◁	◁	◁	▶
26	▶	▶	▶	–	▶	¤	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	¤	▶	▶
27	◁	¤	¤	–	¤	◁	¤	◁	▶	◁	◁	¤	◁	◁	◁	◁	¤	¤	◁	◁	▶
28	¤	▶	–	◁	¤	◁	¤	◁	–	–	¤	▶	◁	¤	–	◁	▶	▶	◁	◁	▶
30	–	▶	▶	–	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	–	–	▶	–	▶	▶
31	▶	▶	▶	–	▶	◁	¤	–	▶	▶	–	¤	–	◁	▶	–	▶	▶	◁	¤	▶
32	–	▶	▶	–	▶	◁	¤	◁	▶	◁	◁	▶	–	◁	▶	–	◁	–	◁	¤	▶
33	–	▶	–	–	–	▶	–	▶	▶	▶	▶	▶	–	▶	–	–	▶	▶	▶	▶	▶
34	–	–	–	◁	–	◁	◁	¤	–	–	¤	–	–	–	◁	¤	◁	▶	◁	◁	–
35	◁	¤	◁	◁	¤	◁	◁	◁	▶	–	◁	◁	◁	◁	◁	–	◁	◁	◁	◁	¤
36	◁	¤	–	◁	–	◁	–	◁	▶	◁	◁	◁	◁	◁	◁	◁	¤	◁	◁	¤	▶
37	–	▶	▶	–	▶	¤	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	◁	▶	▶
38	–	▶	▶	–	▶	–	▶	◁	▶	▶	◁	▶	–	◁	▶	–	¤	▶	◁	▶	▶
39	▶	–	–	◁	▶	–	▶	▶	▶	▶	◁	▶	–	–	–	¤	–	▶	–	–	▶
41	▶	▶	–	–	▶	◁	¤	◁	▶	▶	◁	▶	–	◁	–	▶	–	▶	◁	–	▶
42	◁	◁	–	◁	–	◁	◁	◁	▶	¤	◁	▶	–	◁	–	◁	◁	–	◁	◁	▶
43	▶	▶	▶	▶	▶	¤	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	¤	▶	▶
45	–	¤	¤	◁	–	◁	◁	–	▶	–	▶	▶	◁	◁	–	–	◁	▶	◁	¤	▶
46	▶	–	–	◁	▶	◁	◁	◁	–	▶	–	–	–	◁	–	◁	–	–	–	–	–

Table 6: Part I of the "more risk averse than" table for the symmetric treatment with $\pi = (½, ½)$ at individual SSD-AEI-level. A ◁ indicates that the row subject is revealed more risk averse than the column subject, ▶ indicates that the column subject is revealed more risk averse than the row subject, and ¤ indicates that neither of the subjects is partially revealed more risk averse to the other. A "–" indicates that both subjects are partially more risk averse than the other. Subject numbers correspond to subject IDs 201-219 and 301-328, i.e. number 20 has ID 301 etc. Data from Choi et al. (2007a).

Symmetric Treatment: Part II

	24	25	26	27	28	30	31	32	33	34	35	36	37	38	39	41	42	43	45	46
2	–	➤	➢	➤	¤	–	➢	–	–	–	➤	➤	–	–	➢	➢	➤	➢	–	➢
3	–	➤	➢	¤	➢	➢	➢	➢	➢	–	¤	¤	➢	➢	–	➢	➤	➢	¤	–
4	–	–	➢	¤	–	➢	➢	➢	–	–	➤	–	➢	➢	–	–	–	➢	¤	–
5	–	➤	–	–	➤	–	–	–	–	➤	➤	➤	–	–	➤	–	➤	➢	➤	➤
6	–	➤	➢	¤	¤	➢	➢	➢	–	–	¤	–	➢	➢	➢	➢	–	➢	–	➢
7	➤	➤	¤	➤	➤	➢	➤	➤	➢	➤	➤	➤	¤	–	–	➤	➤	¤	➤	➤
8	–	➤	➢	¤	¤	➢	¤	¤	–	➤	➤	–	➢	➢	➢	¤	➤	➢	➤	➤
9	–	➤	➢	➤	➤	➢	–	➤	➢	¤	➤	➤	➢	➤	➢	➤	➤	➢	–	➤
10	–	➢	➢	➢	–	➢	➢	➢	➢	–	➢	➢	➢	➢	➢	➢	➢	➢	➢	–
12	–	➤	➢	➤	–	➢	➢	➤	➢	–	–	➤	➢	➢	➢	➢	¤	➢	–	➢
13	–	➤	➢	➤	¤	➢	–	➤	➢	¤	➤	➤	➢	➤	➤	➤	➤	➢	➢	–
14	–	–	➢	¤	➢	➢	¤	➢	➢	–	➤	➤	➢	➢	➢	➢	➢	➢	➢	–
15	–	–	➢	➤	➤	➢	–	–	–	–	➤	➤	➢	–	–	–	–	➢	➤	–
16	–	➤	➢	➤	¤	➢	➤	➤	➢	–	➤	➤	➢	➤	–	➤	➤	➢	➤	➤
17	–	–	➢	➤	–	➢	➢	➢	–	➤	➤	➤	➢	➢	–	–	–	➢	–	–
18	–	➤	➢	➤	➤	–	–	–	–	¤	–	➤	➢	–	¤	➢	➤	➢	–	➤
19	–	➤	➢	¤	➢	–	➢	➤	➢	➤	➤	¤	➢	¤	–	–	➤	➢	➤	–
20	–	➤	➢	¤	➢	➢	➢	–	➢	➢	➤	➤	➢	➢	➢	➢	–	➢	➢	–
21	–	➤	¤	➤	➤	–	➤	➤	➢	➤	➤	➤	➤	➤	–	➤	➤	¤	➤	–
22	–	➤	➢	➤	➤	➢	¤	¤	➢	➤	➤	¤	➢	➢	–	–	➤	➢	¤	–
23	–	➢	➢	➢	➢	➢	➢	➢	➢	–	¤	➢	➢	➢	➢	➢	➢	➢	➢	–
24	•	–	➢	–	–	–	➤	–	–	➢	–	–	➢	–	–	–	–	–	–	–
25	–	•	➢	➢	–	➢	➢	➢	➢	➢	➤	➢	➢	➢	➢	➢	➢	➢	➢	–
26	➤	➤	•	➤	➤	➢	➤	➤	➤	➤	➤	➤	➤	➤	–	➤	➤	➢	➤	➤
27	–	➤	➢	•	➤	➢	¤	¤	➢	–	➤	¤	➢	➢	➢	➢	–	➢	¤	–
28	–	–	➢	➢	•	➢	➤	¤	➢	¤	➤	➤	➢	–	➢	➢	–	➢	–	➢
30	–	➤	➤	➤	➤	•	➤	➤	➤	–	➤	➤	➤	➤	–	➤	➤	➤	➤	–
31	➢	➤	➢	¤	➢	➢	•	➤	➢	➤	➤	¤	➢	➢	–	–	➤	➢	¤	–
32	–	➤	➢	¤	¤	➢	➢	•	➢	–	➤	➤	➢	➢	–	➢	➤	➢	¤	–
33	–	➤	➢	➤	➤	➢	➤	➤	•	–	➤	➤	➤	➤	–	➤	➤	➤	➤	–
34	➤	➤	➢	–	¤	–	➢	–	–	•	➢	¤	➢	–	¤	➢	–	➢	–	➢
35	–	➢	➢	➢	➢	➢	➢	➢	➢	➤	•	➢	➢	➢	➢	➢	➢	➢	➢	➢
36	–	➤	➢	¤	➢	➢	¤	➢	➢	¤	➤	•	➢	➢	➢	➢	➤	➢	¤	➢
37	➤	➤	➢	➤	➤	➢	➤	➤	➢	➤	➤	➤	•	➤	–	➤	➤	¤	➤	–
38	–	➤	➢	➤	–	➢	➤	➤	➢	–	➤	➤	➢	•	–	–	➤	➢	¤	–
39	–	➤	–	➤	➤	–	–	–	–	¤	➤	➤	–	–	•	–	➤	➢	–	➤
41	–	➤	➢	➤	➤	➢	–	➤	➢	➤	➤	➤	➢	–	–	•	➤	➢	–	–
42	–	➤	➢	–	–	➢	➢	➢	➢	–	➤	➢	➢	➢	➢	➢	•	➢	–	➢
43	–	➤	➤	➤	➤	➢	➤	➤	➢	➤	➤	➤	¤	➤	➤	➤	➤	•	➤	➤
45	–	➤	➢	¤	–	➢	¤	¤	➢	–	➤	¤	➢	¤	–	–	–	➢	•	–
46	–	–	➢	–	➤	–	–	–	–	➤	➤	➤	–	–	➢	–	➤	➢	–	•

Asymmetric Treatment: Part I																				
	1	2	3	4	5	7	8	9	10	11	13	14	15	16	17	18	19	20	22	23
1	•	◁	¤	▶	▶	–	¤	▶	▶	◁	–	¤	◁	▶	¤	¤	▶	¤	◁	◁
2	▶	•	◁	▶	–	–	–	▶	▶	◁	–	▶	–	–	▶	▶	▶	◁	–	–
3	¤	▶	•	–	–	–	–	–	–	¤	–	¤	¤	–	¤	▶	–	¤	▶	¤
4	◁	◁	–	•	▶	–	–	◁	◁	◁	–	◁	◁	◁	◁	◁	◁	◁	◁	◁
5	◁	–	–	◁	•	–	–	◁	–	◁	–	◁	◁	◁	◁	¤	–	◁	◁	¤
7	–	–	–	–	–	•	◁	◁	◁	◁	◁	–	–	◁	–	◁	–	–	◁	–
8	¤	–	–	–	–	▶	•	◁	¤	◁	–	▶	–	¤	◁	–	▶	–	–	–
9	◁	◁	–	▶	▶	▶	▶	•	▶	◁	▶	¤	–	¤	¤	¤	▶	◁	◁	–
10	◁	◁	–	▶	–	▶	¤	◁	•	◁	–	◁	–	¤	–	–	▶	◁	–	–
11	▶	▶	¤	▶	▶	▶	▶	▶	▶	•	▶	▶	–	▶	▶	▶	▶	▶	▶	▶
13	–	–	–	–	–	▶	–	◁	–	◁	•	–	–	–	–	▶	–	◁	–	–
14	¤	◁	¤	▶	▶	–	◁	¤	▶	◁	–	•	–	◁	¤	¤	▶	◁	◁	◁
15	▶	–	¤	▶	▶	–	–	–	–	–	–	–	•	–	▶	▶	–	–	–	▶
16	◁	–	–	▶	▶	▶	¤	¤	¤	◁	–	▶	–	•	◁	¤	▶	◁	–	–
17	¤	◁	¤	▶	▶	–	▶	¤	–	◁	–	¤	◁	▶	•	¤	▶	¤	¤	¤
18	¤	◁	◁	▶	¤	▶	–	¤	–	◁	◁	¤	◁	¤	¤	•	▶	¤	◁	◁
19	◁	◁	–	▶	–	–	◁	◁	◁	◁	–	◁	–	◁	◁	◁	•	◁	–	◁
20	¤	▶	¤	▶	▶	–	–	▶	▶	◁	▶	▶	–	▶	¤	¤	▶	•	▶	–
22	▶	–	◁	▶	▶	▶	–	▶	–	◁	–	▶	–	–	¤	▶	–	◁	•	▶
23	▶	–	¤	▶	¤	–	–	–	–	◁	–	▶	◁	–	¤	▶	▶	–	◁	•
24	¤	◁	◁	▶	¤	–	–	◁	–	◁	–	◁	◁	◁	¤	¤	▶	¤	–	◁
26	◁	◁	–	▶	–	–	–	◁	◁	◁	–	◁	–	◁	◁	¤	¤	◁	–	–
27	◁	◁	◁	▶	¤	▶	–	◁	▶	◁	–	¤	–	–	◁	¤	▶	◁	◁	◁
28	◁	–	–	▶	◁	¤	▶	◁	▶	◁	–	–	–	¤	◁	¤	–	◁	◁	–
29	◁	◁	◁	▶	▶	▶	–	◁	–	◁	–	¤	–	–	◁	▶	▶	◁	◁	–
30	◁	◁	–	¤	¤	▶	–	◁	–	◁	–	¤	◁	◁	¤	¤	▶	◁	◁	◁
31	¤	–	◁	▶	▶	▶	▶	▶	▶	◁	▶	▶	–	▶	▶	▶	▶	–	–	–
32	◁	–	–	–	–	¤	◁	◁	◁	◁	◁	◁	–	◁	–	◁	–	◁	◁	–
33	–	–	–	–	–	¤	◁	◁	–	◁	◁	◁	–	◁	–	¤	–	◁	◁	–
34	–	¤	¤	▶	▶	–	–	–	–	¤	–	◁	◁	–	¤	–	–	¤	–	¤
36	–	–	–	–	–	▶	▶	◁	▶	◁	▶	–	–	–	–	–	▶	◁	–	–
37	▶	▶	¤	▶	▶	▶	▶	▶	▶	¤	–	▶	◁	▶	▶	▶	▶	¤	▶	▶
38	▶	–	¤	▶	▶	▶	–	▶	▶	¤	–	–	¤	▶	▶	▶	▶	▶	▶	▶
39	¤	◁	◁	▶	–	–	–	▶	¤	◁	–	¤	–	–	◁	¤	–	◁	–	◁
41	◁	–	◁	▶	▶	▶	▶	¤	▶	◁	▶	¤	–	◁	◁	◁	–	▶	¤	–
42	¤	◁	◁	–	◁	–	–	–	–	◁	–	◁	◁	–	◁	◁	–	◁	◁	◁
43	¤	◁	–	–	¤	–	–	¤	▶	◁	–	¤	◁	◁	◁	◁	▶	◁	¤	◁
44	▶	◁	–	–	▶	–	◁	◁	◁	◁	◁	◁	◁	◁	◁	¤	–	◁	◁	◁
46	–	–	¤	–	¤	–	–	▶	▶	◁	–	¤	◁	–	¤	▶	–	¤	▶	▶

Table 7: Part I of the “more risk averse than” table for the asymmetric treatment with $\pi = (⅓, ⅔)$ at individual Ssd-Aei-level. Subject numbers correspond to subject IDs 401-417, 501-520, and 601-609, i.e. number 18 has ID 501, number 38 has ID 601, etc.

Asymmetric Treatment: Part II																			
	24	26	27	28	29	30	31	32	33	34	36	37	38	39	41	42	43	44	46
1	⌑	▶	▶	▶	▶	▶	⌑	▶	–	–	–	◁	◁	⌑	▶	⌑	⌑	◁	–
2	▶	▶	▶	–	▶	▶	–	–	–	⌑	–	◁	–	▶	–	▶	▶	▶	–
3	▶	–	▶	–	▶	–	▶	–	–	⌑	–	⌑	⌑	▶	▶	▶	–	–	⌑
4	◁	◁	◁	◁	◁	⌑	◁	–	–	◁	–	◁	◁	◁	◁	–	–	–	–
5	⌑	–	⌑	▶	◁	⌑	◁	–	–	◁	–	◁	◁	–	◁	▶	⌑	◁	⌑
7	–	–	◁	⌑	◁	◁	◁	⌑	⌑	–	◁	◁	◁	–	◁	–	–	–	–
8	–	–	–	◁	–	–	◁	▶	▶	–	◁	◁	–	–	◁	–	–	▶	–
9	▶	▶	▶	▶	▶	▶	◁	▶	▶	–	▶	◁	◁	◁	⌑	–	⌑	▶	◁
10	–	▶	◁	◁	–	–	◁	▶	–	–	◁	◁	◁	⌑	◁	–	◁	▶	◁
11	▶	▶	▶	▶	▶	▶	▶	▶	▶	⌑	▶	⌑	⌑	▶	▶	▶	▶	▶	▶
13	–	–	–	–	–	–	◁	▶	▶	–	◁	–	–	–	◁	–	–	▶	–
14	▶	▶	⌑	–	⌑	⌑	◁	▶	▶	▶	–	◁	–	⌑	⌑	▶	⌑	▶	⌑
15	▶	–	–	–	–	▶	–	–	–	▶	–	▶	⌑	–	–	▶	▶	▶	▶
16	▶	▶	–	⌑	–	▶	◁	▶	▶	–	–	◁	◁	–	▶	–	▶	▶	–
17	⌑	▶	▶	▶	▶	⌑	◁	–	–	⌑	–	◁	◁	▶	▶	▶	▶	▶	⌑
18	⌑	⌑	⌑	⌑	◁	⌑	◁	▶	⌑	–	–	◁	◁	⌑	▶	▶	▶	⌑	◁
19	◁	⌑	◁	–	◁	◁	◁	–	–	–	◁	◁	◁	–	–	–	◁	–	–
20	⌑	▶	▶	▶	▶	▶	–	▶	▶	⌑	▶	⌑	◁	▶	◁	▶	▶	▶	⌑
22	–	–	▶	▶	▶	▶	–	▶	▶	–	–	◁	◁	–	⌑	▶	⌑	▶	◁
23	▶	–	▶	–	–	▶	–	–	–	⌑	–	◁	◁	▶	–	▶	▶	▶	◁
24	•	–	–	▶	◁	▶	◁	–	–	◁	–	◁	◁	▶	–	◁	▶	▶	◁
26	–	•	◁	◁	◁	⌑	◁	–	◁	–	◁	◁	◁	◁	–	–	◁	◁	–
27	–	▶	•	▶	▶	⌑	◁	▶	▶	◁	–	◁	◁	◁	▶	–	◁	▶	–
28	◁	▶	◁	•	–	⌑	◁	⌑	▶	–	◁	◁	–	–	◁	–	◁	◁	–
29	▶	▶	◁	–	•	▶	◁	–	–	⌑	–	◁	◁	▶	◁	–	▶	▶	⌑
30	◁	⌑	⌑	⌑	◁	•	◁	◁	◁	–	–	◁	◁	◁	–	–	◁	⌑	–
31	▶	▶	▶	▶	▶	▶	•	▶	▶	–	▶	◁	◁	▶	▶	▶	▶	▶	▶
32	–	–	◁	⌑	–	▶	◁	•	⌑	–	◁	◁	◁	◁	◁	–	–	–	–
33	–	▶	◁	◁	–	▶	◁	⌑	•	–	◁	◁	–	◁	◁	–	–	▶	–
34	▶	–	▶	–	⌑	–	–	–	–	•	–	◁	⌑	–	–	▶	–	–	⌑
36	–	▶	–	▶	–	–	◁	▶	▶	–	•	◁	◁	▶	◁	–	–	–	–
37	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	•	◁	▶	▶	▶	▶	▶	▶
38	▶	▶	▶	–	▶	▶	▶	▶	–	⌑	▶	▶	•	▶	–	▶	▶	▶	▶
39	◁	▶	▶	–	◁	▶	◁	▶	▶	–	◁	◁	◁	•	–	⌑	▶	⌑	◁
41	–	–	◁	▶	▶	–	◁	▶	▶	–	▶	◁	–	–	•	–	⌑	▶	–
42	▶	–	–	–	–	–	◁	–	–	◁	–	◁	◁	⌑	–	•	–	–	◁
43	◁	▶	▶	▶	◁	▶	◁	–	–	–	–	◁	◁	◁	⌑	–	•	–	–
44	◁	▶	◁	▶	◁	⌑	◁	–	◁	–	–	◁	◁	⌑	◁	–	–	•	–
46	▶	–	–	–	⌑	–	◁	–	–	⌑	–	◁	◁	▶	–	▶	–	–	•

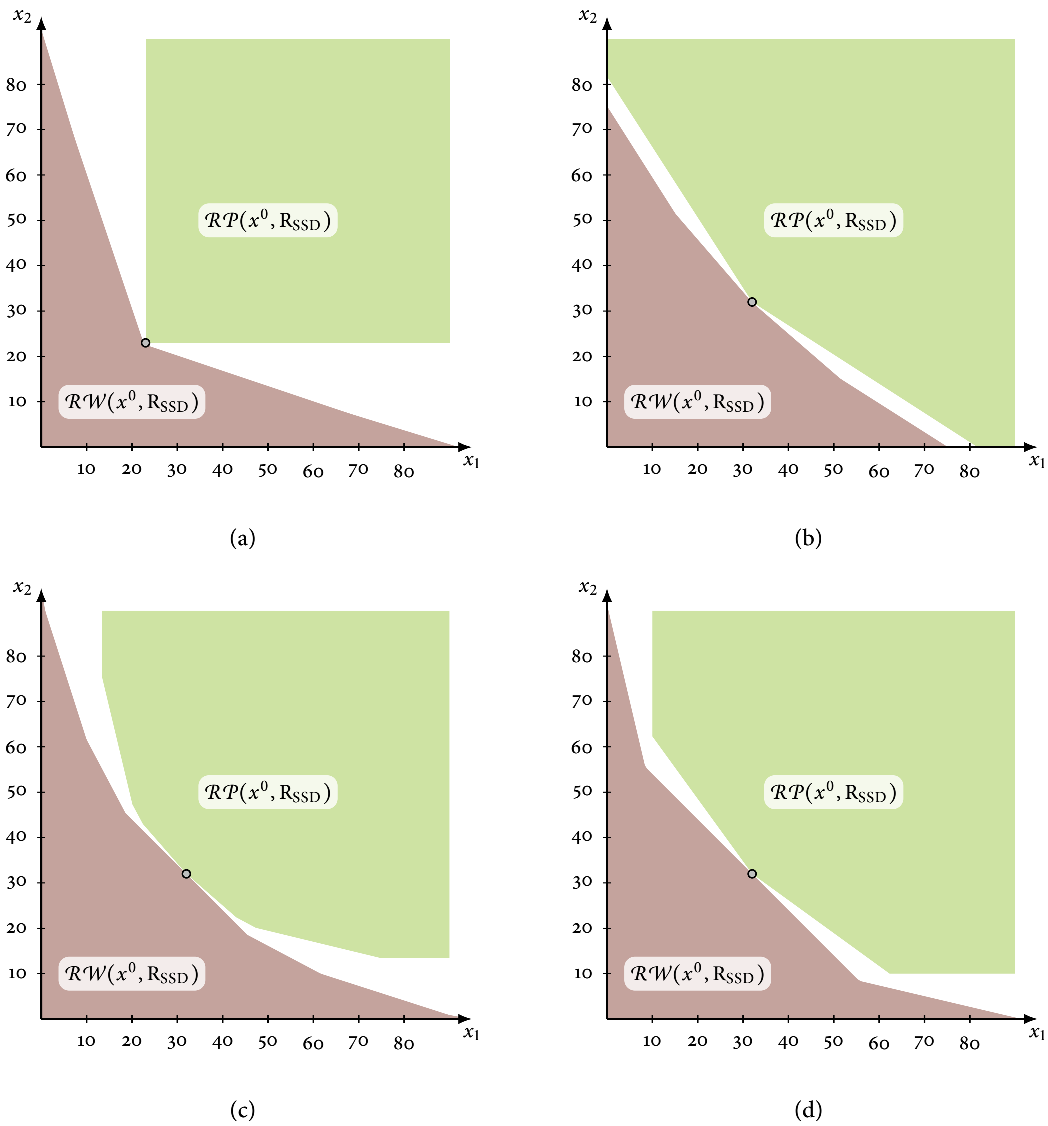

Figure 8: Examples of subjects' revealed preferred and revealed worse sets, from the symmetric treatment. (a) Subject number 23 (ID 304): A subject who is revealed more risk averse than most other subjects. (b) Subject number 26 (ID 307): A subject who is revealed less risk averse than most other subjects. (c) Subject number 8 (ID 208): A subject who is revealed more risk averse and revealed less risk averse than some other subject and has similar preferences as many other subjects. (d) Subject number 5 (ID 205): A subject who is incomparable with some other subjects. Data from Choi et al. (2007a).

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